THE

THEORY OF PERSPECTIVE

DEMONSTRATED.

THE THEORY OF PERSPECTIVE DEMONSTRATED;

IN A METHOD ENTIRELY NEW.

By which the ſeveral

PLANES, LINES, AND POINTS,

MADE USE OF IN THIS ART,

Are ſhewn in the true Poſitions in which they are to be conſidered.

Invented, and now publiſhed for the Uſe of

The ROYAL ACADEMY at WOOLWICH.

BY JOHN LODGE COWLEY,

PROFESSOR OF MATHEMATICKS.

LONDON,

Sold by J. BENNETT, in Crown-Court, St. Ann's, Soho; Mathematical Inſtrument-Maker to their Royal Highneſſes WILLIAM Duke of Glouceſter, Prince HENRY, and Prince FREDERICK.

MDCCLXV.

TO THE MOST HONOURABLE

JOHN MANNERS,

MARQUIS OF GRANBY,

One of His Majesty's Most Honourable Privy Council,
Lord Lieutenant Custos Rotulorum of the County of Derby,
Colonel of the Royal Regiment of Horse Guards,
Lieutenant General of His Majesty's Forces, and
Master General of the Ordnance, &c. &c. &c.

MY LORD,

THE honour [illegible] employed in [illegible] Your [illegible] of my endeavours to facilitate the theory of the art of PERSPECTIVE, warms me to the most faithful discharge of [illegible], and every other duty of my station; well knowing it to be my best recommendation to Your Lordship's future favour and protection.

I am, with perfect esteem,

My Lord,

Your Lordship's most obedient,

Royal Academy, Woolwich,
[illegible] and most devoted humble Servant,

John Lodge Cowley.

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MY LORD,

THE great Honour Your Lordship has condescended to bestow upon me in Your acceptance of my endeavours to facilitate the theory or true idea of PERSPECTIVE, warms me to the most faithful discharge of THIS, and every other duty of my station; well knowing it to be my best recommendation to Your Lordship's future favour and protection.

I am, with perfect esteem,

MY LORD,

Your Lordship's most obedient,

And most devoted humble servant,

John Lodge Cowley.

Royal Academy, Woolwich,
May 1, 1765.

PREFACE.

I shall not here point out the usefulness of PERSPECTIVE, or the general [illegible] I have endeavoured to explain, since the [illegible] writings of very eminent persons, employed on this subject, are sufficient testimony.

[illegible] endeavours to facilitate [illegible] PERSPECTIVE, [illegible] the easiest [illegible] well knowing it to be my [illegible] knowledge, planned and improved by others, [illegible] and to convey the sense of my Lectures to Students, in the most easy and perspicuous manner, is the main [illegible]

The quick sale of the whole impression of my [illegible] DIX. TO EUCLID, and the repeated demands for that treatise, are indubitable marks of publick approbation; and have determined me to expedite a second edition of

P R E F A C E.

I Shall not here point out the uſefulneſs of PERSPECTIVE, or the ſuperior excellence of the principles I have endeavoured to explain, ſince the numerous writings of very eminent perſons, employed on this ſubject, are ſufficient teſtimonies of the former; and it is hoped the enſuing Treatiſe will fully evidence the latter. On the other hand, I frankly own, that my performance allows me not any reaſonable claim or pretence to an enlargement of ſcience, by extending its dominions, though this is confeſſedly the chief merit of an author. Inſtructing youth in certain branches of mathematical knowledge, planned and improved by others, is the province to which I am devoted; and to convey the ſenſe of my Lectures to ſtudents, in the moſt eaſy and perſpicuous manner, is the main point I aſpire to.

The quick ſale of the whole impreſſion of my APPENDIX TO EUCLID, and the repeated demands for that treatiſe, are indubitable marks of publick approbation; and have determined me to expedite a ſecond edition of

it, or rather a new and more comprehensive work, which I believe will prove a considerable help to such as engage in that part of mathematical learning; no less persuaded that this my present attempt, to explain and facilitate the doctrine of Planes as applicable to Perspective, will have a similar effect, especially as it has been approved of by THE HONOURABLE SIR CHARLES FREDERICK, SURVEYOR GENERAL OF HIS MAJESTY'S ORDNANCE, who was pleased to consider it in manuscript; on which foundation I, with the more confidence, submit it to the publick.

J. L. C.

A SUCCINCT

HISTORY OF PERSPECTIVE.

PERSPECTIVE, according to the accounts of hiſtorians, is derived from painting, particularly as employed in theatrical decorations, but was very little known until about the beginning of the ſixteenth century. The deſire of repreſenting on a plane, or flat ſurface, ſuch figures as ſhould produce, on the eyes of the ſpectators, effects ſimilar to thoſe which would be occaſioned by their being ſeen in relief, and at different diſtances, engaged the artiſts of thoſe times to conſider the apparent diminution of magnitude and alterations of poſition, which ſuch different objects exhibit to the eye, according as they are nearer to or farther from the ſame.

In this reſearch it is, at leaſt, very probable, that a due obſervance of natural appearances furniſhed the firſt hints to intelligent painters how to proceed in this part of their art; for, by looking at a range of objects placed on lines parallel to each other, as rows of trees, &c. they could not but ſee them as converging together, and appearing nearer and nearer to each other in proportion to their remoteneſs: the ground, although really level, would ſeem as riſing upwards with a gentle aſcent; on the other hand, in viewing a level plane elevated on high, as a flat cieling, &c. they would ſee it as appearing to decline or ſink in going off from the eye.

Theſe, and ſuch like notices, may reaſonably be ſuppoſed the primary or principal guides by which artiſts were directed to make the imitations which they firſt produced of theſe kinds of appearances; and that illuſions of this ſort being publickly exhibited, induced geometricians,

who are not to be ſatisfied with any thing leſs than rigorous exactneſs, to examine ſtrictly into the cauſes of theſe effects, and the means of accurately inveſtigating perfect imitations of theſe illuſive appearances, and thereby Perſpective is become a ſyſtem of mathematical rules, the true guide and firm ſupport of the imitative arts.

The ancients made it a general and leading principle to conſider the objects, which they would repreſent, as being beyond a tranſparent plane, ſo placed between the eye and the objects, that all rays iſſuing from them to the eye ſhould paſs through the ſame, and impreſs thereon ſuch images as would have the ſame effect to an eye placed in the proper point for viewing them, as the real objects themſelves would produce when ſeen in their natural ſtate; and the moderns have aſſented to this poſition of the ancients; for, ſeeing that the image or repreſentation, thus imprinted on that plane, ſends to the eye the ſame rays as the real object itſelf would do, it is manifeſt, from all that has been diſcovered in optics, that no other ſenſation can be produced, but that ſimilar effects will be wrought upon the ſpectator; and that, if the variations of magnitude and poſition be duly aſſiſted by the art of colouring, the illuſion will be compleat: whence the art of Perſpective conſiſts abſolutely in a geometrical determination of the points in which the ſeveral rays cut the ſaid tranſparent plane in proceeding from each point of the given objects to the eye. In ſhort, a perſpective repreſentation is no other than a projection of objects, due regard being had to the poſition or place of the eye.

This fundamental principle of the ancients may be yet more generally extended; for it is not neceſſary to imagine the objects beyond a tranſparent plane, nor even that the plane ſhould be tranſparent; for the objects may be conſidered and treated as being between the eye and the plane on which they are to be repreſented,

and the viſual rays, by which the objects are perceived, may be produced till they meet the plane which is to exhibit their images, and thereby mark out other points analogous to thoſe of the original objects from whence they proceeded, which ſeveral rays will form on that plane, when thus poſited, correſpondent images, which will, in like manner, ſhew the perſpective repreſentations of the given objects; but the other way of applying this principle is moſt generally uſed, and is therefore particularly alluded to in def. I. page 17. nevertheleſs the geometrician is at liberty to have recourſe to either, as ſhall be moſt convenient.

VITRUVIUS, in his Architecture, book VII. chapter I. has preſerved ſome of the Perſpective of the ancient Greeks; he obſerves, that one AGATARCHUS, having been inſtructed by ESCHYLUS how to draw theatrical decorations, was the firſt who wrote upon the ſubject; that Agatarchus taught his art to DEMOCRITUS and ANAXAGORAS, and that thoſe two, who were geometers, had alſo wrote about it. Vitruvius adds, that they ſhewed how to draw lines from a point in a certain place, ſo as ſhall aſſuredly repreſent buildings in a decoration, in ſuch manner, that ſome of them ſhall appear to project or advance towards the eye, while others at the ſame time ſeem to fall back and retire.

In this manner does the Roman architect explain the performances of Democritus and Anaxagoras; we need only be barely initiated in the rudiments of optics to enable us for diſtinguiſhing the reſemblance which the principles of the ancient Perſpective have to thoſe of our own; for the point in a certain place, is that we call the point of ſight, or place of the eye, which determines the poſition of almoſt all the lineaments of the object.

This is all we have left concerning the Perſpective of the ancients, as far as I can find; ſo that we may, at leaſt, fairly account the mo-

derns, the ſecond inventors of this art, which took its riſe among us in thoſe glorious days of painting, the latter end of the fifteenth, or the beginning of the ſixteenth century.

Two artiſts, who were alſo good geometricians, ALBERT DURER in Germany, and PIETRO DEL BORGO SAN STEPHANO in Italy, gave rules for putting objects in perſpective. Durer did it mechanically by the help of a machine, whoſe construction and uſe are founded upon the ancient principle before-mentioned. Borgo, who was a little before Durer, wrote three books upon the ſubject, which Ignatio Dante highly extols, but they are loſt to us.

After this author DANIEL BARBARO, patriarch of Aquileia, publiſhed a treatiſe on this art, in 1579.

BALTHAZAR PERUZZI, of Sienna, was happy in clearing Perſpective of many incumbrances which it laboured under in Italy, and gave it a degree of elegance, by introducing the uſe of points different from thoſe before made uſe of, as known to us, which are called POINTS OF DISTANCE: this author has been exactly followed, even to a degree of ſervility, by VIGNOLA, the famous Italian architect, in his Treatiſe of Perſpective. SIRIGATTI made uſe of Vignola in the ſame manner; and ANDREA POZZO, ſo lately as the year 1700, indulged himſelf in a free uſe of the ſame liberty.

GUIDO UBALDI MARQUIS DE MONTE, in a folio treatiſe, printed at Peſaro in 1600, conſidered Perſpective in a more ſcientific view than any of the former; he appears as being the firſt whoſe ideas tended towards rendering the principles of Perſpective univerſal, and greatly improved the art, by advancing this very prolific principle, viz. that all lines, parallel to one another and to the horizon, although inclined to the table, or picture as it is now called, would conſtantly converge towards a point in the horizontal line, and that

the ſaid point is that where the ſaid horizontal line is interſected by the line drawn from the eye parallel to the afore-mentioned lines.

Ubaldi might indeed have made his principle more general, by ſhewing, that all lines parallel to one another, though not parallel to the horizon, do meet in one certain point of the picture, namely, in that where it is cut by that particular parallel ray which is drawn from the eye; and there are conditions which require this augmentation to what he advanced.

But what Ubaldi has given us may be made to ſuffice in the ordinary caſes of Perſpective, where ſuch objects moſt generally preſent themſelves as are terminated by lines perpendicular or parallel to the horizon; for it is plain, that the apparent concourſe of all lines perpendicular to the plane of the picture, being in the principal point, is only a particular caſe of Ubaldi's principle; for the ſaid principal point is no other than that in which the picture is interſected by the perpendicular drawn to the eye. In like manner, lines inclined 45 degrees to the plane of the picture, meet in that point of the horizontal line in which it is cut by a line, drawn from the eye under an angle of 45 degrees. All parallels, inclined 30 degrees to the plane of the picture, will appear to concur in that point thereof, in which a line, drawn from the eye under the like angle, meets the ſame; and the like of others. So that it would be no difficult matter to ſolve the general and fundamental problem of all Perſpective after 25 different ways, as Ubaldi has done, but the ſame may be performed by other methods innumerable. As for the reſt of Ubaldi's work, it has the fault common to others of that time of day, and what he has there expanded, through a multitude of propoſitions, might have been elegantly comprehended in the compaſs of a few pages.

With respect to what has been transmitted to us by Marolois, the Jesuit, and others, whom we call the writers on the old Perspective, in contradistinction to the system we here endeavour to explain, it seems sufficiently manifest, that the works of Peruzzi and Ubaldi have been the general store-house to which those several writers have had recourse for the principles they make use of; we shall therefore pass over, for the present, the numerous writings which have appeared on this part of the mathematicks, and not make any farther excursions on a subject, whose greatest difficulties are within the reach of a moderate geometrician, but proceed to shew what were the next advances made towards bringing it to a state of perfection, and into general use among artists, and others desirous of being acquainted with this useful and pleasing art of deception.

The celebrated geometrician, Dr. Brook Taylor, F. R. S. observing how confined, intricate and ungeometrical, the principles of Perspective were in his time, how few, simple and universal they in reality might be, desirous of establishing new principles of simple construction, universal in application, and supported by geometrical demonstrations, condescended to write on this subject, and published a small tract, under the title of LINEAR PERSPECTIVE, in the year 1715.

In this little piece the doctor made prodigious advances towards bringing this art to its ultimate degree of perfection. He justly observed, that all planes, considered as such, are alike in geometry, and should in like manner be applied to Perspective; that there are no exclusive honours due to the ground plane, nor any particular magic in the horizontal plane, nor consequently in their correspondent lines or points. He relieved us from the contracted limits in which our conceptions of this subject were inscribed, enlarged and extended our ideas to universality itself, taught us the true use of vanishing

planes, lines and points, in all ſituations in which they can poſſibly be conceived, whether parallel, perpendicular, or any how inclining to the picture, or to the original objects, &c.

But finding that many objected againſt it on account of its not being ſufficiently eaſy to be generally underſtood; he therefore, in the year 1719, publiſhed a ſecond ſmall treatiſe, called NEW PRINCIPLES OF LINEAR PERSPECTIVE.

But this likewiſe, on account of the mathematical dreſs and brevity of expreſſion in which it was delivered, was not ſo generally careſſed as it deſerved to be, nor the principles it contained ſo much examined as he wiſhed; wherefore he deſigned the publick another treatiſe, propoſing therein to ſet forth thoſe principles in another light, ſuch that their preheminence above all others then in uſe might be more readily perceived, and the whole better adapted to the conceptions of young artiſts applying themſelves to Perſpective for works of deſign; but his death happening before the completion of thoſe intentions, the world was thereby deprived of the advantage of ſuch a perfect piece as might reaſonably be expected from his great abilities, and this method of Perſpective remained a knowledge enjoyed by few, and thoſe chiefly ſuch in whoſe hands it was not of the greateſt utility to the publick, through their not being of employments which required ſkill in thoſe principles, for making the proper applications of them required in the arts of deſign.

In the year 1738, JOHN HAMILTON, Eſq; F. R. S. publiſhed two volumes, folio, under the title of STEREOGRAPHY, or a compleat body of Perſpective in all its branches, the projections of ſhadows, reflexions by poliſhed planes, &c. in ſeven books. This learned and ingenious author hath very copiouſly treated the ſubject of Perſpective in a ſtrict mathematical way, with the aſſiſtance of Dr. Taylor's principles, whom he acknowledges to have made, in a few pages

only, more real advances towards perfecting the ſcience of Perſpective, than all the writers who went before him. In this treatiſe, and alſo in thoſe wrote by MOXON, POZZO, MAROLOIS, LEYBOURN, and many others, he may ſee his miſtake, who aſſerts that it hath not been ſhewn how to find the perſpectives upon concave or convex ſurfaces, nor upon a figure of ſeveral faces, much leſs by reflexion or refraction, &c.* But this treaſury of mathematical projection is little attended to, as being too difficult and diffuſive for general uſe among artiſts, notwithſtanding it is worthy of being conſulted occaſionally in ſome extraordinary caſes that may chance to occur, and has been conſidered in what is here offered the reader.

In the year 1754, Mr. Kirby, deſigner in perſpective to their preſent MAJESTIES, publiſhed a treatiſe in quarto, titled Dr. Brook Taylor's Method of Perſpective made eaſy, &c. and in the year 1755, a ſecond edition thereof; which D. Fournier thought worthy for him to make free uſe of in his publication in the year 1764.

In 1761, he publiſhed the Perſpective of Architecture, a large and elegant work in folio, containing two rules of univerſal application; improvements in the doctrine of ſhadows, the deſcription and uſe of a new and very uſeful inſtrument, called the architectonic ſector; and the preſent year 1765 produces the third edition of that firſt publiſhed in 1754.

This ingenious author, by attentively examining and applying Dr. Taylor's new principles of Perſpective to practice, was gradually led to a diſcovery of their generality and facility in operation, ſaw how preferable and excellent they were in practical applications, how

* See Elements of Mathematicks, &c. together with a new Treatiſe of Perſpective, for the uſe of the Royal Academy at Woolwich. Printed for and ſold by J. Milan, Bookſeller, oppoſite the Admiralty, Charing-croſs, 1765.

ſimple and extenſive their conſtructions, what a vaſt confuſion of unneceſſary lines were thereby avoided, and how beneficial they would be if generally known to artiſts concerned in works of deſign; poſſeſſed with theſe and ſuch like conſiderations, he employed himſelf zealouſly to retrieve them from that ſtate of darkneſs in which their author's brevity of expreſſion and manner of writing had concealed them, and became the firſt among artiſts, who appeared in publick, to explain their true nature and uſe in adapting them ſuitably to the arts of deſign.

The encouragement he met with at his firſt communicating this deſign, how joyfully our artiſts in general embraced it, the ſucceſs which attended and overcame all the various oppoſitions he had to encounter in the proſecution of it, would come now to be explained, were I to continue the thread of my hiſtorical narrative beyond this period of time: but as theſe are things which would lead me to tranſmit an impartial account of ſome of this author's inventions, yet in manuſcript, as alſo to recite the exploded ſyſtems again revived, the controverſal writings produced thereupon, and other ſuch like attempts, made for continuing theſe new principles in their former obſcurity, which, being matters that I think not altogether proper to be explicitly handled in this co-temporary publication, I reſerve them to a future ſeaſon, and paſs over this part of its hiſtory in ſilence, for the ſake of preventing any undue ſuggeſtions which my perſonal acquaintance with Mr. Kirby might, perhaps, be the means of bringing upon the moſt fair and juſt account that can be given, ſhall therefore confine myſelf to only one general and well-atteſted fact, publickly declared to all, and which ſhould alſo, for the very ſame reaſon, have been here omitted, had it not appeared to me, that the juſtice due to the memory of thoſe ingenious authors, who have contributed to improve and exalt this art to perfection and ge-

neral utility, required my endeavours to rescue them from the undeserved censure lately passed upon them, of not having made the least improvement *; as well as to vindicate ourselves, by shewing that what we have advanced in favour of our author is not our own bold assertion only, but has the united suffrage of a body of artists, well qualified to judge decisively in this matter, by whose order the following paragraph was inserted in the several publick papers of the year 1754, viz.

ACADEMY of PAINTING and SCULPTURE, in St. Martin's-Lane. Jan. 24, 1754.

MR. KIRBY, author of a work, intituled, DR. BROOK TAYLOR'S METHOD OF PERSPECTIVE, MADE EASY, &c. has read three lectures (being the substance of his intended work) to the gentlemen of this society, which appeared to them so clear, simple and extensive, that, in order to do justice to so excellent a performance, they have unanimously given this their public approbation, and declared the ingenious author an honorary member of their body.

By order,
F. M. NEWTON, Secretary.

To conclude, I shall now only mention a few more of the many writings we have on this subject, viz.

HONDIUS, his perspective institutions were formerly held in great esteem.

ALLEAUME'S deserves to be more known than it is, being well adapted to the purposes of artists.

FATHER DE CHALES'S is remarkably neat.

S'GRAVESANDE'S ESSAI DE PERSPECTIVE, published at Leiden in 1711, contains many new things, and is recommendable for practice.

* See page 311 of the Treatise before alluded to.

FATHER LAMY's contains ſome proper notices on the ſubject of painting.

M. DE LA CAILLE's performance deſerves to be noticed with reſpect.

THE NEW TREATISE, referred to in the preceeding note, is diſtinguiſhable for its extreme brevity and peculiar ſingularities, &c.

As to what I have here done, my whole deſign has been to lay before learners the method I have uſed in my private courſe of communicating the knowledge of thoſe principles ſo far as concerns one of my profeſſion. I do not pretend to exhibit a new treatiſe of perſpective, my pretence is to render the art rationally underſtood, facilitate the ſtudy of it, point out the true principles that ought to be applied by thoſe who would arrive at perfection in it, and to ſhew them how to diſtinguiſh between true principles and falſe ones, in an art where deception loſes all its beauty, when ſo groſſly handled as not to bear a correſponding reſemblance to reality.

ADVERTISEMENT.

In page 7, line 6, inſtead of SAME FIGURE, read as follows, viz. Plate I. Fig. 1. Make the planes B M, S H, interſect each other in the line H L.

In page 11, let the ſecond line begin thus, viz.
Raiſe up the plane D Y, paſs the plane S H through the ſame, and make the plane B M paſs through the plane S H.

Whenever lines are mentioned, they are to be underſtood as being ſtraight lines.

The references here made uſe of allude to the Elements of Euclid, as publiſhed by Profeſſor Simſon, of Glaſgow, octavo edition, 1762, thus (4. 1.) denotes the fourth propoſition of the firſt book, &c.

THE

THEORY OF PERSPECTIVE

DEMONSTRATED.

PART I.

OF THE DOCTRINE OF PLANES.

DEFINITIONS.

I.

A Line is perpendicular, or at right angles to a plane, when it makes right angles with every line meeting it in that plane.

ILLUSTRATION. Plate I. Fig. 1.

Make F in the plane F B coincide with F in the plane F N. Then, if the angles E D F, E D K, E D N, &c. are each a right angle, the line E D is perpendicular to the plane F N.

II.

A plane is perpendicular to a plane, when all lines drawn in one of the planes, perpendicularly to the common section of the two planes, are perpendicular to the other plane.

SAME FIGURE.

The lines B A, E D, G F, &c. being drawn in the plane B F perpendicularly to A F, the common ſection of the two planes B F, F N, if they are alſo perpendicular to the plane F N, the plane B F paſſing through thoſe lines, is perpendicular to the plane F N.

III.

The inclination of a line to a plane is the acute angle formed by that line, and another line drawn from the point, in which the firſt line meets the plane, to the point in which a perpendicular to the plane, drawn from any point of the firſt-mentioned line above the plane, meets the ſame plane.

SAME FIGURE.

Thus the acute angle F D G is the inclination of the line D G to the plane F N.

IV.

The inclination of a plane to a plane is the acute angle formed by two lines drawn from any the ſame point of their common ſection at right angles to it, one upon one plane, and the other upon the other plane.

SAME FIGURE.

Raiſe up the planes D Y, Z X, making W Y coincide with W y. Then the line D W, being the common ſection of the planes D Y, D Z, and R T perpendicular thereto, and R S alſo perpendicular thereto, at the ſame point R, the acute angle S R T is the inclination of the plane D Y to the plane D Z.

V.

Two planes are ſaid to have the ſame or a like inclination one to the other, which two other planes have to each other, when the ſaid angles of inclination are equal to one another.

VI.

Two planes, which, being either way produced, do not meet each other, are ſaid to be parallel one to the other.

VII.

A pyramid is a ſolid figure contained by planes that are conſtituted betwixt one plane and one point out of that plane, all meeting in that one point.

ILLUSTRATION. Plate II. Fig. 2.

Make B in the triangle L A B coincide with B in the plane L B, alſo make C coincide with C, and L with L, and B in the triangle B A E with B in the plane L B, then will the Figure thus formed repreſent a pyramid.

VIII.

The point A is the vertex of the pyramid. The plane L B the baſe thereof. The planes L A B, B A C, &c. the Sides thereof. And the line A H the perpendicular altitude or height thereof above the plane of its baſe L B.

THEOREM I.

One part of a line cannot be in a plane and another part thereof above it.

PLATE I. Fig. 1.

Make F in the plane F B coincide with F in the plane F N.

DEMONSTRATION.

For, if it be poſſible, let A D, part of the line A D G, be in the plane F N, and the part D G above the ſame, then A D being in the plane F N, it can be produced in that plane ſuppoſe to F; now the points D and G are in the plane B F, which paſſes through the

line AF, whence the line DG is in the plane BF, (7 def. 1 .) wherefore the two lines ADG, ADF, are in the ſame plane, and have a common ſegment AD, which is impoſſible (cor. 2. 1.)

Q. E. D.

THEOREM II.

Two lines, which cut one another, are in one plane, and three lines, which meet one another, are in one plane.

SAME FIGURE.

Make F coincide with F, and the planes BM, SH paſs through each other in the line HL, and let p k, r H be two lines cutting one another in C; I ſay thoſe two lines, as alſo the three lines CH, C k, H k, meeting each other, are in one plane.

DEMONSTRATION.

For neither can p C, part of p k, nor r C, part of r H, be in one plane, and the other parts CH, C k, out of that plane by the precedent; wherefore the lines p k, r H are in one plane. And if you ſay that CP x, part of the triangle CH k, is in one plane, and the other part PH k x in another plane, then muſt CP be in one plane and PH in another, the ſame likewiſe of C x and x k, which is impoſſible; therefore the triangle CH k is in one plane. *Q. E. D.*

THEOREM III.

If two planes interſect each other, their common ſection is a ſtraight line.

SAME FIGURE.

Let H and L be the extremes of the common ſection of the two planes BM, SH, join thoſe extremes by drawing the line HL.

DEMONSTRATION.

Then is the line H L in both the ſaid planes; that line is therefore their common ſection by conſtruction, for if you ſay it is not in both the planes, then may two lines be drawn between the ſame extremes, one from L to H in the plane S H, and another from L to H in the plane B M; thus would two lines, bounded between the ſame extremes, include a ſpace, which is impoſſible, (axiom 10. 1.) wherefore H L muſt neceſſarily be in both the planes S H, B M, which is therefore their common ſection. *Q. E. D.*

THEOREM IV.

If one line be perpendicular to the common ſection of two other lines which interſect each other, it is alſo perpendicular to the plane which paſſes through the ſaid two interſecting lines.

ILLUSTRATION. Plate II. Fig. 3.

Make E in the plane E B C to coincide with E in the plane C D E F, and paſs the plane F B D through the plane E B C, alſo make the plane G B H paſs through both the ſaid planes, making H coincide with H.

DEMONSTRATION.

Let there be taken A C = A D = A E = A F, and draw the lines CD, C F, F E, E D alſo through A in the plane C D E F, draw the line G H any how, meeting the lines C F, D E, in G and H, and let A B be perpendicular to the two lines C E, D F at their common ſection A, and draw the lines B C, B F, B E, B H BG and B D.

Now by conſtruction A C, A D, A E and A F are equal to each other, and the angle C A F = D A E, (15. 1.) whence C F = D E (4. 1.) and the angle F C A or G C A = D E A or H E A, ſo like-

C

wiſe G A C=H A E, and A C=A E; therefore is A G=A H and G C=H E (26. 1.)

Again, becauſe A B is perpendicular to the plane C D E F by hypotheſis, the triangles C A B, D A B, E A B, F A B, are each right angled at A, having their baſes equal each to each, and the perpendicular A B common to them all, whence their hypothenuſes will alſo be equal each to each, B C=B D=B E=B F; wherefore the triangles C B F, D B E, being mutually equilateral, the angle F C B or G C B=D E B or H E B, (8. 1.) and becauſe G C=H E and B C=B E, therefore is B G=B H.

But it was proved above, that A G=A H, and A B is common, whence the angle G A B=H A B, conſequently A B is perpendicular to the line G H (def. 10. 1.) and in the ſame manner it may be proved, that A B is perpendicular alſo to C E, F D, and all other lines whatever that can be drawn in the plane C D E F through the point A, on which it inſiſts; therefore A B is perpendicular to the plane paſſing through the lines C E, D F, &c. *Q. E. D.*

COROLLARY.

Hence it follows, that when a line, as A B, is at right angles to ſeveral lines, as A F, A E, &c. which it meets in the ſame point, as A, the lines which it ſo meets, are all in the ſame plane; for a line, as A K, drawn from A out of the plane C D E F, cannot be perpendicular to A B, becauſe the angle B A K will be leſs or greater than a right angle, or B A H, according as A K is drawn above or below the ſaid plane C D E F.

THEOREM V.

If in a given plane a line be drawn through any point thereof, and two other lines perpendicular thereto be alſo drawn from the ſaid

given point, one of them in the ſaid given plane, and the other in any other plane paſſing through the firſt-mentioned line, I ſay, that a line drawn from the ſaid given point at right angles to the firſt of thoſe two perpendiculars, will alſo be perpendicular to the given plane at the given point.

SAME FIGURE.

Let B M be the given plane, C the given point, H L a line drawn through the ſame, C E a line drawn in the given plane perpendicular to H L, and C R another line perpendicular to H L, but drawn in ſome other plane, as S H, paſſing through the firſt-mentioned line H L; alſo let C I be a line drawn from the given point C at right angles to the firſt perpendicular C E, and it will alſo be perpendicular to the given plane B M at the given point C.

DEMONSTRATION.

Becauſe C H is perpendicular both to C E and C R by hypotheſis, it will alſo be perpendicular to C I by the preceedent, and therefore C I being perpendicular to C E as well as to C H by hypotheſis, it will alſo be perpendiculor to the plane B M, which contains the lines C H and C E, by the preceedent. *Q. E. D.*

THEOREM VI.

Two lines, which are perpendicular to one and the ſame plane, are parallel to each other.

SAME FIGURE.

Make K in the triangle G F K coincide with K in the plane F N, and let E D and G F be each perpendicular to the ſame plane F N,

and in that plane draw the line F D, and perpendicular thereto draw D K=F G, alſo let there be drawn the lines F K, G K and G D.

DEMONSTRATION.

The triangles G F D, F D K, having G F=D K, by conſtruction, the ſide F D common, and the angle G F D=F D K, each being a right angle, will alſo have G D=F K, (4. 1.) whence the triangles G D K, G F K being mutually equilateral, the angle G D K or G F K is a right angle, (8. 1.) but the line E D is alſo perpendicular to D K, as well as the lines G D, F D; it is therefore in the ſame plane with them E D F G, conſequently the angles G F D, E D F being right angles, E D is therefore alſo parallel to F G. *Q. E. D.*

COROLLARY.

Hence it appears, that there cannot be drawn more than one perpendicular from the ſame point to one and the ſame plane, for all lines perpendicular to the ſame plane are parallel to each other, which lines, drawn from one and the ſame point, cannot be.

THEOREM VII.

If there be two parallel lines, one of which is perpendicular to a plane, the other will alſo be perpendicular to that ſame plane.

SAME FIGURE.

The angles F D K, G D K being right angles, and G D in the ſame plane with the propoſed parallels G F, E D, the angle E D K is alſo a right angle, (4. 11.) as is alſo the angle E D F; therefore is E D perpendicular to the plane F D K. *Q. E. D.*

THEOREM VIII.

If a line be drawn from any point in one of two parallel lines to any point in the other, that line will be in the ſame plane with the ſaid parallels.

SAME FIGURE.

Let B X, E Q be two parallel lines in the plane B M, and take any point H in one of them, and any point C in the other, then will the line which joins the points H and C, be in the plane of the propoſed parallels.

DEMONSTRATION.

If you ſay it is not, ſuppoſe it above that plane as the line C u H, and in the plane of the parallels draw C H, then will the lines C u H, C H, be terminated by the ſame extremes, and include a ſpace, which is impoſſible, (ax. 10. 1.) whence the line joining the points C and H can not be above the plane of the propoſed parallels, in like manner it will appear that it can not be below the ſame; therefore it muſt be in the ſame plane with them. *Q. E. D.*

THEOREM IX.

If a line be perpendicular to a plane, any plane paſſing through that line, will be at right angles to the plane whereto the ſaid line is perpendicular.

SAME FIGURE.

Let E D be a line perpendicular to the plane F N, and B F a plane paſſing through that line, and it will alſo be perpendicular to the ſaid plane F N.

DEMONSTRATION.

From any point, as B, taken at pleaſure in the plane B F, draw

B A perpendicular to A F, the common ſection of the two planes B F, F N, then is the angle F A B=F D E, equal a right angle, (def. 3. 11.) whence B A is parallel to E D, and is therefore perpendicular to the plane F N, by the foregoing theorem 7. In like manner it may be proved, that every line which can be drawn in the plane B F perpendicularly to A F, the common ſection of the planes, will be alſo perpendicular to the plane F N, therefore the plane BF, paſſing through thoſe lines, will itſelf be alſo perpendicular to the plane F N. *Q. E. D.*

COROLLARY I.

Hence it follows, that the plane F N, according to the ſenſe of the definition, is perpendicular to the plane B F; for let D K be a line drawn in the plane F N, perpendicular to A F, the common ſection of both planes, at the point D, and it will alſo be perpendicular to E D; and therefore the plane F N, in which it was ſo drawn, is alſo perpendicular to the plane B F, paſſing through the ſaid line E D.

COROLLARY II.

Hence it alſo follows, that a line, drawn at right angles to one of two perpendicular planes from any point of their common ſection, will be in that other plane. For let the two perpendicular planes be F N, B F, their common ſection A F, and D a point in the ſame; alſo let D E be at right angles to the plane F N, then muſt D E be in the plane B F, for no other line can be drawn from the point D at right angles to the plane F N, as appears by the corollary to the foregoing theorem 6.

THEOREM X.

Planes, to which one and the ſame line is perpendicular, are parallel to each other.

SAME FIGURE.

Let E C be perpendicular to the plane B F, and alſo to the plane S H, then are thoſe planes parallel.

DEMONSTRATION.

From any point, as D, in the plane B F, draw D R parallel to E C, and it will alſo be perpendicular to both the planes B F, S H, by theorem 7. Let the lines E D, C R be drawn, now ſeeing the angles at E, C, R and D, are each right ones, (def. 1.) the figure E C R D is a rectangled parallelogram, by reaſon its ſides E D, C R, are in the ſame plane D Y, with the parallels E C, D R, conſequently E C = D R, the ſame way it can be proved, that all other perpendiculars, terminated by thoſe two planes B F, S H, are alſo equal to each other; therefore the ſaid planes are parallel. *Q. E. D.*

COROLLARY.

Hence it appears, that all lines, which are perpendicular to one of two parallel planes, are alſo perpendicular to the other.

SCHOLIUM.

From the two laſt theorems, the ſenſe and propriety of the two definitions of perpendicular and parallel planes appear manifeſt.

THEOREM XI.

Lines which are parallel to one and the ſame line, though not in the ſame plane with them, are parallel to each other.

SAME FIGURE.

Paſs the plane S H through the plane D Y, alſo paſs the plane B M

through the planes S H and Z X, and let B G, X M, be each parallel to Z W then are they alſo parallel to each other.

DEMONSTRATION.

Draw E W, W Q, each perpendicular to ZW, then will W Q be perpendicular to the plane paſſing through E W Q, and E B, Q X, will alſo be perpendicular to the ſame plane, whence they are alſo parallel to each other; therefore the lines B G, X M, are parallel one to the other. *Q. E. D.*

THEOREM XII.

If two lines, meeting each other in one plane, be reſpectively parallel to two other lines alſo meeting each other in ſome other plane, then will the angle, formed by the two lines meeting in the firſt-mentioned plane, be equal to that formed by the meeting of the other two lines in the other plane.

SAME FIGURE.

Let the lines L C, C R, in the plane S H, be reſpectively parallel to the lines G E, E D, in the plane B F, then will the angle L C R be equal to the angle G E D.

DEMONSTRATION.

Take C L, C R, G E, E D, all equal to one another, and let C E, R D, D G, L R, G L, be drawn, then C R and D E, being equal and parallel to each other, as alſo L C equal and parallel to G E by hypotheſis and conſtruction, G L and R D are both equal and parallel to C E, conſequently equal and parallel to each other, (33. 1.) whence D G=L R, and the triangles L C R, G E D, be-

ing mutually equilateral, the angle L C R is therefore equal to the angle G E D (8. 1.) *Q. E. D.*

THEOREM XIII.

If two lines, meeting each other in one plane, be reſpectively parallel to two other lines, meeting each other in another plane, then the two planes, paſſing through thoſe lines, will alſo be parallel to each other.

SAME FIGURE.

Let the lines E D, E G, in the plane B F, be reſpectively parallel to the lines U c, U O, in the plane S H, then are the planes B F, S H, paſſing through thoſe lines, parallel to each other.

DEMONSTRATION.

Draw E C perpendicular to the plane B F, meeting the plane S H in C, in which plane let there be alſo drawn C r, r ſ, reſpectively parallel to U O, U c, then are C r, r ſ, reſpectively parallel to E G, E D, by theorem 11, and becauſe C E G, C E D, are right angles by conſtruction, E C L, E C R, are likewiſe right angles; whence E C, being perpendicular to the plane S H, by theorem 4, and alſo to the plane B F, by conſtruction, the two planes B F, S H, are therefore parallel one to the other, by theorem 10. *Q. E. D.*

THEOREM XIV.

If two parallel planes are cut by a third plane, the ſections made thereby are parallel to each other.

SAME FIGURE.

Let the parallel planes B F, S H, be cut by the plane D Y, then are the ſections E D, C R, parallel one to the other.

DEMONSTRATION.

Draw E L and D S parallel to each other, alſo draw E C, D R, perpendicular to the plane S H, and draw C L, S R, then E L being parallel to D S, by conſtruction, and E C parallel to D R, by theorem 6, the angle C L E = R S D, by theorem 12, but the angle E C L = D R S, as being each a right angle, and E C = D R; whence E L is both equal and parallel to D S, therefore E D is alſo parallel to C R. *Q. E. D.*

COROLLARY.

Hence it appears, that lines parallel to each other, and terminated by the ſame parallel planes, are alſo equal to each other.

THEOREM XV.

If, from the extremes of a line paſſing through a given plane, be drawn two lines, each meeting the ſaid plane at right angles, the line, which joins the two points where thoſe two perpendicular lines meet the ſaid plane, will be cut by the firſt-mentioned line, in that point where it paſſes through the ſaid plane, and be thereby divided into two ſuch parts, as bear the ſame ratio to each other, which the two perpendiculars, meeting the plane, have one to the other.

SAME FIGURE.

Let n m be the line cutting the given plane B M in the point C, and from n and m, the extremes thereof, draw the perpendiculars n r, m H, meeting the plane S H in the points r and H, and draw the line H r; then will r C : C H :: r n : H m.

DEMONSTRATION.

Produce n r as here to ſ, then ſeeing n ſ, m H, are each perpendicular to the plane B M, by hypotheſis, they are parallel to each

other by theorem 6, whence the lines n m, H r, being both in the ſame plane with thoſe parallels, and terminated by them, as they are not parallel to each other, they will meet or interſect each other, and thereby make the alternate angles equal one to the other, that is, r n C = H m C, as alſo r C n = H C m; whence the triangles n r C, m H C, being ſimilar, it will therefore be r C : C H :: n r : m H. *Q. E. D.*

COROLLARY.

Hence it follows, that if, in the plane B M, there be drawn the lines r f, H X, parallel to each other, and in them be taken r p = n r, and H k = H m, then will the line p k, joining the points p and k, interſect H r in the ſelf ſame point in which it was before interſected by the line n m. For if h be conceived as the interſection point of p k with H r, the triangles p r h, H h k, will be equiangular; whence we have r h : h H :: r p : H k,

and n r : H m :: r C : C H,

but r p = n r, and H k = H m,

wherefore n r : H m :: r C : C H; therefore, ſeeing r H is divided into one and the ſame ratio by both the points C and h, thoſe two points muſt neceſſarily coincide together, or vaniſh into one point only.

THEOREM XVI.

If two planes, interſecting each other, be both perpendicular to a third plane, the common ſection of thoſe two planes will be alſo perpendicular to the ſaid third plane.

SAME FIGURE.

Let the planes D Y, S H, be each perpendicular to the plane A Z, interſecting each other in the line C R, then is their common ſection C R perpendicular to the plane A Z.

DEMONSTRATION.

From R, one extreme of the common section, draw a line, as R C, perpendicular to the plane A Z, then this perpendicular, being in both the planes D Y, S H, by cor. 2 to theorem 9, it must necessarily be their common section, which is therefore perpendicular to the plane A Z, by construction. *Q. E. D.*

PART II.

OF THE DOCTRINE OF PLANES APPLIED TO THE TRUE PRINCIPLES OF PERSPECTIVE.

DEFINITIONS.

I.

PERSPECTIVE is that part of mathematical projection which gives rules for describing, upon any given plane, the representations of any given objects, so as to exhibit thereon the exact forms, magnitudes and positions thereof, such as they would be seen to have upon that plane, by an eye fixed in one certain point, and viewing them from thence through the said given plane, supposing it to be perfectly transparent, or as glass, &c.

ILLUSTRATION. Plate III. Fig. 4.

Raise up the plane D Y, and pass the plane S H through the same, in the line C P, then bring R, in the plane F G, to coincide with R in the plane F M, raise up the plane M X, and pass the plane G X through both the planes S H and M X.

Suppose now the plane S H transparent, and A B a line in the plane F M, seen by an eye fixed at E, and viewing it through the plane S H, then will A and B, the extremes of that line, appear on the plane S H, in the points a and b, where the lines A E, B E, drawn from, or issuing thence to the eye at E, cut or intersect the said plane S H; and all other lines which can be drawn, or conceived to issue from any other of the points in the line A B to the eye at E, will, by their intersections with the plane S H, mark out other intermediate points between a and b, and the whole line A B

F

will be thereby depicted, on the plane S H, by the line a b, which is therefore the true description or representation thereof.

SCHOLIUM.

Hence it appears, that the given plane, on which the representations of the given objects are to be drawn, must be so situated in respect to the given objects and the point in which the eye is placed, that all lines, which can be drawn or conceived to issue from them to the eye, may pass through the said plane; for, seeing the representations thereof are determined upon that plane by the intersections made thereon, by lines passing through it in proceeding from them to the eye, it is plain there can not be any representations formed on that plane of objects which are so posited, that the lines issuing from them to the eye, do not cut the plane on which they are required to be described; whence the necessity of the above-mentioned position of the said given plane appears manifest.

II.

The plane S H, upon which the given objects are represented, is, in mathematical terms, called the plane of projection, but here we term it the PICTURE.

III.

The plane, which contains the objects given to be described on the picture, is called the ORIGINAL PLANE.

Thus the plane F M is here the original plane.

IV.

The point, in which the eye is fixed for viewing the given objects, or their representations on the picture, is called the place of the eye, or POINT OF SIGHT.

Thus E is the point of sight.

SCHOLIUM.

The place of the eye, or point of ſight, ought to be juſtly aſcertained, by having due regard to the ſize, ſituation, &c. of the picture and original plane; for an injudicious determination in this reſpect, produces ill effects in the repreſentations formed on the picture, as appears from a recent inſtance hereof in a late production, in which the author has made ample diſplay of inconſiſtent repreſentations, formed by tranſgreſſing the rules requiſite to be obſerved on this occaſion. How faults of this kind are to be avoided, will be ſhewn in the annotations or general comment hereto annexed.

V.

A line drawn from the eye, perpendicular to the picture, and meeting the ſame, is called the DISTANCE, OR AXIS OF THE EYE, or diſtance of the picture.

Thus E C is the diſtance or axis of the eye, or diſtance of the picture.

VI.

The point, in which the axis of the eye meets the picture, is the CENTER OF THE PICTURE.

Thus C is the center of the picture S H.

VII.

The deſcriptions made upon the picture of any original objects, whether they be points, lines, ſurfaces or ſolids, are called the perſpective repreſentations or IMAGES thereof.

Thus the line a b is the perſpective repreſentation or image of the given line A B.

VIII.

A plane, paſſing through the point of ſight, parallel to any original plane, is called the VANISHING PLANE of that original plane.

Thus the plane G X, being parallel to the original plane F M, and paſſing through E, the point of ſight, is the vaniſhing plane of the ſaid plane F M.

IX.

A plane, paſſing through the point of ſight, parallel to the picture, is called the DIRECTING PLANE.

Thus the plane F G, being parallel to the picture S H, and paſſing through E, the point of ſight, is the directing plane.

X.

A plane, paſſing through the axis of the eye at right angles to the original plane, is the VERTICAL PLANE of the ſaid original plane.

Thus D Y is the vertical plane of the original plane F M.

XI.

The line, in which the directing plane cuts the original plane, is called the DIRECTING LINE of that original plane.

Thus F R is the directing line of the original plane F M.

XII.

The line, in which a vaniſhing plane cuts the picture, is called the VANISHING LINE of the original plane correſponding to that vaniſhing plane.

Thus H L is the vaniſhing line of the original plane F M.

XIII.

The line, in which a vaniſhing plane cuts the directing plane, is called the EYE'S PARALLEL.

Thus I G is the parallel of the eye when placed in E.

XIV.

The line, in which the vertical plane interſects the directing plane, is called the DIRECTOR, OR HEIGHT OF THE EYE.

Thus E D is the director or height of the eye when fixed in E.

XV.

The line, in which the picture intersects the original plane, is called the INTERSECTING LINE of that plane.

Thus S P is the intersecting line of the original plane F M.

XVI.

The line, in which the picture intersects the vertical plane, is called the VERTICAL LINE of the original plane.

Thus C P is the vertical line of the original plane F M.

XVII.

The point, in which an original line, produced when necessary, cuts the intersecting line, is called the INTERSECTING POINT of that original line.

Thus P is the intersecting point of the original line A B, and S that of the original line M N.

XVIII.

The point, in which an original line, produced if necessary, cuts the directing line, is called the DIRECTING POINT of that original line.

Thus D is here the directing point of A B, and also of M N.

XIX.

The point, in which a line, drawn from the point of sight, parallel to any original line, cuts the picture, is called the VANISHING POINT of that original line.

Thus E C, being drawn parallel to A B, the point C is the vanishing point of A B; and E V, being parallel to M N, the point V is the vanishing point of that line M N.

XX.

The point, in which the vertical line cuts a vanishing line, is called the CENTER OF THAT VANISHING LINE.

Thus C is the center of the vaniſhing line H L, the ſame here with that of the picture.

XXI.

The diſtance between the point of ſight and the center of a vaniſhing line, is called the DISTANCE OF THAT VANISHING LINE.

Thus E C is the diſtance of the vaniſhing line H L.

XXII.

The diſtance between the point of ſight, and the point in which a line drawn from thence, parallel to any original line, cuts the picture, is called the DISTANCE OF THE VANISHING POINT of that original line.

Thus E C is the diſtance of C, the vaniſhing point of A B, and E V, the diſtance of V, the vaniſhing point of M N.

XXIII.

When the image on the picture is a point, the line which produces it in paſſing from the original to the eye, is called a RAY, or OPTIC RAY, &c.

Thus A E, producing the image a, is called a ray, as is alſo B E, which produces the image b.

XXIV.

When the image is a line, the ſeveral rays which produce it are together called a PLANE OF RAYS.

Thus all the rays, that can be conceived to iſſue from each of the points in A B, meeting together in E, conſtitute a plane of rays.

XXV.

When the original is a circle, then the ſeveral rays, which produce its image on the picture, are called a CONE OF RAYS.

XXVI.

And when the original is a right-lined ſurface or ſolid, the rays,

issuing from the several points thereof, are called a PYRAMID OF RAYS.

ILLUSTRATION. Plate IV. Fig. 5.

Raise up the plane D C, and make the plane O P pass through the same, then raise up the plane S T, and bring D, in the plane P D, to coincide with D in the plane D K, and pass the plane L B through both the planes O P, S T, also bring D, in the plane D Y, to coincide with D, and pass the plane M N through both the planes O P, S T.

Then the three planes of rays B E C, C E G, B E G, taken together, form a pyramid of rays, and so of the others, as exhibited in the figure.

THEOREM I.

The parallel of the eye, intersecting, vanishing, and directing lines, belonging all to the same original plane, are parallel to each other.

PLATE III. Fig. 4.

Make the picture S H pass through the vertical plane D Y in the line C P, and pass the plane G X through both the planes S H and M X.

Now let F M be the original plane, I G the parallel of the eye, S P the intersecting, H L the vanishing, and F R the directing lines thereof; I say, those several lines are parallel one to the other.

DEMONSTRATION.

Because the picture and directing plane are parallel, (def. IX.) the vanishing and original planes also parallel, (def. VIII.) the sections made by those planes are parallel; (theorem XIV. part I.) therefore the lines I G, S P, H L and F R, are parallel to each other.

Q. E. D.

COROLLARY.

Hence it follows, that any one of thoſe lines, and any point in another of them, being given, that other line may thence be determined; for a line, drawn through that point, parallel to the given line, will be the line required. But it does not follow, as an axiom, that theſe two lines will be in the ſame plane, for, before that can be rightly aſſerted, the interpoſition of another plane, diſtinct from the former, muſt be introduced.

THEOREM II.

The vertical plane is perpendicular to the picture, the vaniſhing, directing and original planes, and alſo to the parallel of the eye, the interſecting, vaniſhing, and directing lines of that ſame original plane.

SAME FIGURE.

Let D Y be the vertical plane, and the other mentioned planes and lines the ſame as in the preceedent.

DEMONSTRATION.

The line E C is perpendicular to the picture, (def. V.) and the vertical plane D Y paſſes through that line, (def. X.) the vertical plane is therefore alſo perpendicular to the picture, (theorem IX. part I.) and likewiſe to the directing plane, which is parallel thereto. (def. IX.) the ſaid vertical plane is alſo perpendicular to the original plane, (def. X.) whence it is alſo perpendicular to the vaniſhing plane, that being parallel thereto; (def. VIII.) wherefore the ſaid vertical plane, being perpendicular to each of the four above-mentioned planes, it is therefore perpendicular to the common ſections I G, H L, S P, F R. *Q. E. D.*

THEOREM III.

The director is perpendicular both to the directing line and the parallel of the eye. And the vertical line is perpendicular both to the intersecting and vanishing lines.

SAME FIGURE.

Let E D be the director, C P the vertical line, and the rest as before.

DEMONSTRATION.

The lines I G, H L, S P, F R, being all perpendicular to the plane D Y by the preceedent, they are therefore perpendicular to all lines meeting them in that plane. (def. I. part I.) Now, seeing E C, C P, P D, D E, are in the plane D Y, therefore is E D perpendicular to F R and I G; also C P perpendicular to H L and S P, consequently it is as the theorem enounces. *Q. E. D.*

THEOREM IV.

Original planes, parallel to the picture, have neither vanishing, intersecting, nor directing lines, neither have the lines, situated in those planes, any of those corresponding points.

SAME FIGURE.

Let M X be an original plane parallel to the picture S H, and Z X an original line situated in that plane; I say, the plane M X hath neither vanishing, intersecting, or directing lines, nor hath the line Z X any of them points.

DEMONSTRATION.

For the original plane M X, being parallel to the picture, can not cut it to determine the intersecting line as is required, (def. XV.) the

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vaniſhing plane F G being alſo, in this caſe, parallel to the picture S H, can not determine a vaniſhing line to the original plane M X, (def. XII.) for the like reaſon there can be no directing line, the directing plane F G being here parallel to the original plane M X, as well as to the picture S H.

Again, ſeeing the plane G X drawn through the eye at E and the original line Z X, cuts the picture in H L, and the directing plane in I G, thoſe two lines are parallel, (theorem XIII. part I.) they are alſo parallel to Z X, ſince a plane M X may be drawn through that line parallel to the plane S H; whence I G is the line which ought to produce the vaniſhing point of the line Z X, (def. XIX.) but, being parallel to the picture, it can not. The original line Z X, being alſo parallel both to the picture and directing plane, can cut neither of them; therefore the plane M X and the line Z X, are deſtitute of either vaniſhing, interſecting, or directing lines, or points. *Q.E.D.*

COROLLARY I.

If an original plane M X, parallel to the picture, cut any other plane whatever, as F M, their common ſection A M will be parallel to the vaniſhing, interſecting, and directing lines of the ſaid plane F M.

For the original plane M X, being parallel to the picture and directing plane F G, the ſections A M, S P, F R, made by thoſe three planes, with the plane F M, are parallel. (theorem XIII. part I.) Now S P is the interſecting line, F R the directing line of that plane, H L the vaniſhing line thereof, which is alſo parallel thereto; therefore A M is likewiſe parallel to that vaniſhing line, (theorem X. part I.)

COROLLARY II.

Hence it appears, that the image of an original line, parallel to the picture, is parallel to the ſaid original.

SAME FIGURE.

Let Z X be an original line parallel to the picture S H, and let the plane G X paſs through the point of ſight, and the original line Z X, cutting the picture S H, in the line H L, draw the lines X E, Z E, interſecting the picture in the points W and V; then is the line W V the image of Z X, and they are parallel to each other, by what was proved above.

COROLLARY III.

All parallel original lines, which are parallel to the picture, have parallel images, for thoſe images are parallel to their correſponding originals, by the preceedent corollary.

COROLLARY IV.

Hence it follows, that whatever angle is formed by the meeting of two original lines, which are parallel to the picture, an angle equal thereto will be formed on the picture by the meeting of their correſpondent images.

SAME FIGURE.

Make the planes D Y, D Z, both paſs through the picture S H in the lines C P, V S; raiſe up the plane M X, ſo that Y coincide with Y, and make the plane G X form one continued plane with the plane F E, and let A Y, Y M, be two original lines parallel to the picture, making the angle A Y M, then is the angle P C m, formed on the picture by their correſpondent images C P, C m, equal thereto.

For thoſe images are reſpectively parallel to their originals, (by the preceedent corollary) therefore the angle P C m = A Y M. (theorem XII. part I.) And, univerſally, all lines in an original plane, which is parallel to the picture, are themſelves parallel to the picture, and alſo to their reſpective images.

THEOREM V.

The image of an original line, which is not parallel to the picture, produced both ways, will paſs through both the interſecting and vaniſhing points of the ſaid original line.

SAME FIGURE.

Let now the plane D Y form one continued plane with the plane D M, paſs the plane D Z through the picture S H in the line V S, making D coincide with D and M with M, and paſs the plane G X through the picture. Let now M N be an original line in the original plane F M; S the interſecting, and V the vaniſhing point thereof.

DEMONSTRATION.

Becauſe the lines E V, M N, are parallel, (def. XIX.) all lines that can be drawn from E to the line M N, will be in the ſame plane as the lines E V and M N are in. (7. 11.) Now, ſeeing the points S and V are in this plane, and alſo in the picture, the line V S is the common ſection of thoſe two planes, but n m, the image of N M, is a part of that ſection; therefore n m, produced both ways, will paſs through S and V, the interſecting and vaniſhing points of the original line. *Q. E. D.*

COROLLARY I.

The directing point D can have no image on the picture, by reaſon the line E D, which ought to cut the picture in order to form

an image, is in the directing plane F E; it is therefore parallel to the picture, consequently can not intersect it.

COROLLARY II.

The images of all the points, that can be conceived between the intersecting point S and the extreme point M, how far soever it be produced on from thence, will fall on the picture between the intersecting point S and the vanishing point V; for, wherever the point M be so taken, the angle E M S will be equal to the angle M E V, as being the alternate angles of two parallel lines; wherefore the points m and V can never coincide, nor can the point N be so taken, but that its image will always be between S and V, the angle E N S being, for the like reason, always equal to the angle N E V. Now, when N comes into S, its image n then also comes into the intersecting point S; in like manner, if M be supposed at an infinite distance, so that the angle M E V entirely vanish, and the line E M coincide with E V, then will its image m vanish into the point V. Hence appears the propriety of calling this a vanishing point, as also why the line H L, passing through that point, is termed a vanishing line, and the plane G X, passing through that line, a vanishing plane, &c.

COROLLARY III.

If the original line itself passes through its own vanishing point, its image is then contracted into that point, and the line, in this case, may be said to vanish. Thus, suppose V the vanishing point of N M, and let N M be conceived as moving parallel to itself along the lines N K, M Z, till, being in the position or line Z K, its produced part N S or K V passes through the point V, then is that point only the image of the line N M when in that position, the images n, m, both now coinciding in V.

COROLLARY IV.

Hence it follows, that if an original line be produced and paſs through the point of ſight, the vaniſhing and interſecting points thereof will coincide, and its directing point will be the point of ſight; whence any point in the picture may be the image of any point in the original line paſſing through the eye and that point of the picture, or may be taken as the vaniſhing point of that line, or as the image of its interſection with all planes or lines whatſoever, which it cuts; wherefore, a point being given on the picture, its original can not be thence determined, unleſs the poſition of that original, with reſpect to ſome other known point, line or plane, be alſo given.

THEOREM VI.

Original lines, which are parallel to each other, but not to the picture, have the ſame vaniſhing point, and their images produced all meet in that ſame point.

SAME FIGURE.

Raiſe up the plane D Y, and paſs the picture S H through the ſame; bring R to coincide with R, and make the plane G X paſs through the picture. Let now O Q be parallel to A B, then is C their vaniſhing point, and their images a b, o q, produced, will paſs through the ſame.

DEMONSTRATION.

Becauſe E C is parallel to A B, it will alſo be parallel to O Q; (theorem XI.) wherefore C is their common vaniſhing point. And, by the preceedent, o q, the image of O Q, will, when produced, paſs through the vaniſhing point, and a b, the image of A B, when produced, alſo paſſes through the ſame; therefore the images a b,

o q, of the two original parallel lines A B, O Q, when produced, meet in that ſame vaniſhing point C. *Q. E. D.*

COROLLARY I.

The ſame vaniſhing point can not belong to any two original lines which are not parallel to each other, for the ſame line, drawn from the point of ſight to meet the picture, can not be parallel to more than one of them.

COROLLARY II.

The center of the picture is the vaniſhing point of all lines which are perpendicular to the picture, becauſe the axis of the eye is parallel to all ſuch lines. (def. V. part I. and theorem VII. part II.)

THEOREM VII.

Original lines, whoſe directing points fall in the ſame director, have parallel images, and the angle made at the point of ſight, by the directors of any two original lines, is equal to that formed on the picture by their images.

SAME FIGURE.

Bring D to coincide with D, raiſe up the plane D Y, and make the picture paſs through both the planes D Y and E M; now let N M and O Q be two original lines, having their directing points D and T each in the ſame director E D; then are the images n m, o q, of thoſe two lines, parallel to each other.

DEMONSTRATION.

For the lines C P, V S, are each parallel to the director E D, whence the images n m, o q, being in thoſe lines, are alſo each parallel to the ſame director E D; they are therefore parallel to each

other; and when there are different directors, as all directors meet in the point of ſight, they will alſo be parallel to the images.

Q. E. D.

THEOREM VIII.

If two original lines meet each other, their images will alſo meet each other in that point of the picture, which is the image of that point in which the originals meet one another.

PLATE IV. Fig. 5.

Form the figure according to the directions annexed to definition XXVI.

DEMONSTRATION.

Let B G, A G, be two original lines meeting in G, then are b g and a g the images thereof. Now, ſeeing G is a point common to both the lines A G, B G, the image g of that point will alſo be common to both the images a g, b g; thoſe images therefore meet in the point, which is the image of that point in which the originals met each other. *Q. E. D.*

COROLLARY.

The images of all parallel lines whatever are either parallel, or meet in ſome one point; for, when the originals are parallel to the picture, their images are alſo parallel, and when not parallel to the picture, their images meet in one common vaniſhing point, as was ſhewn above.

THEOREM IX.

All lines in an original plane have their vaniſhing, interſecting and directing points, in the vaniſhing, interſecting and directing lines of that plane.

PLATE III. Fig. 4.

Let the picture now paſs through the plane D Y only, and the plane G X paſs through the picture, and let the plane F M be the original plane, and A B, M N, two given lines in that plane.

DEMONSTRATION.

The lines S P, F R, in which the picture and directing plane are cut by the original plane F M, are the only lines which can be cut by lines drawn in the original plane, as M N, A B, &c. wherefore P and S are the interſecting points of thoſe lines, and D the directing point thereof; for theſe lines meeting, in this caſe, the directing line in the ſame point D, have their directing point common. Now E C, E V, being drawn parallel to A B and M N, determine their vaniſhing points C and V, but theſe lines are alſo parallel to the original plane; wherefore the plane G X, drawn through thoſe lines, will alſo be parallel to the plane containing the lines A B, M N; and as no other plane can be drawn through E parallel thereto, it will be the vaniſhing plane of the original plane, and the line H L the vaniſhing line thereof; but C and V, the vaniſhing points of A B and M N are in that line; therefore the propoſition is manifeſt.

Q. E. D.

COROLLARY I.

The image of any point, line or figure, ſituated in that part of an original plane, which is beyond the interſecting line, muſt fall on the picture between the interſecting and the vaniſhing line, as is evident by what was premiſed above in cor. II. to theorem V.

SCHOLIUM.

From the above it appears, that when the ground is conſidered as the original plane, the vaniſhing line thereof will be the ultimate

extent to which the image of any part of the ground, eſteemed as level, can extend; this line therefore, in that caſe, determines the apparent boundary of the horizon, whence, being taken in this ſenſe, it is called the horizontal line; on the ſame ſuppoſition, the interſecting line is called the ground line, as being then the interſection of the picture with the ground. But if we ſtop here, at this imperfect and confined way of conſidering and applying planes, as adapted by authors of this kind of perſpective, the art will be inſufficient in many caſes, and in general intricate and complex; which a due application of the improvements, furniſhed by the principles here treated of, will render perfect, and make the art of drawing perſpective repreſentations ſimple, eaſy and univerſal.

COROLLARY II.

Any two original lines, in the ſame plane, which are not parallel to the picture, having their images parallel to each other, have a common directing point; for only one director can be drawn parallel to both their images, and that director can cut the directing line of the original plane only in one point; that point is therefore the common directing point of the two original lines. And when any two lines in an original plane, cut the directing line of that plane in the ſame point, their images on the picture will be parallel.

COROLLARY III.

Lines, drawn through any two vaniſhing points, any two interſecting points, and any two directing points, of lines in an original plane, are the vaniſhing, interſecting and directing lines of that plane; for any two points, in a line being given, the whole line is thereby determined.

COROLLARY IV.

All original lines, parallel to an original plane, have their vanishing points in the vanishing line of that plane; for lines may be drawn in the original plane parallel to the proposed lines, and all parallel lines, having the same vanishing points, (theorem VI. part II.) the vanishing points of the proposed lines are therefore in the vanishing line of that original plane to which they are parallel. (theorem IX. part II.)

THEOREM X.

The vanishing line of a plane, perpendicular to the picture, passes through the center of the picture, which it can not do when that plane is in any other position.

DEMONSTRATION.

For the vanishing plane will then pass through the axis of the eye, which is perpendicular to the picture, (def. V. part II.) and all planes, which pass through the axis of the eye, are perpendicular to the picture, wherefore those planes can not be parallel to planes which are not at right angles to the picture; therefore the vanishing planes thereof do not pass through the axis of the eye, consequently the vanishing lines of such planes do not pass through the center of the picture. *Q. E. D.*

THEOREM XI.

All parallel original planes have the same vanishing line, center and axis of the eye, and their intersecting and directing lines are parallel to each other.

PLATE VI. Fig. 7.

Make S, in the plane S L, coincide with S in the plane A K;

make A coincide with A, and the plane G P paſs through the plane S L, in the line R T; raiſe up the plane K L, ſo that the plane G P may paſs through the line U O, and the two lines K P coincide with each other; turn the plane K L about, till K coincides with K, then bring D, in the plane D Y, to coincide with D in the plane A K, and make the plane Y G paſs through the plane S L in the line W X, and paſs the plane F M through the plane K M in the line M V, and paſs the plane V L through the plane S L in the line H L.

Now, let the plane S L repreſent the picture, K M the directing plane, E the point of ſight, A K one original plane, and Y G another parallel thereto, then are the interſecting and directing lines S I, K P, parallel to each other, and both thoſe planes have the ſame vaniſhing line F N, the ſame center c, and axis of the eye E c.

DEMONSTRATION.

For the plane F M, paſſing through E, the point of ſight, parallel to either of the given planes, will alſo be parallel to the other; it is therefore the vaniſhing plane of both the ſaid given planes. (def. VIII. part II.) Now the ſame vaniſhing plane can produce but one vaniſhing line F N, and E c is the axis of the eye belonging thereto, and c the center thereof, and the interſecting and directing lines S I, K P, being parallel to the ſame vaniſhing line F N; they are therefore parallel to each other. (theorem XI. part I.) *Q. E. D.*

COROLLARY I.

Planes, which are not parallel one to another, can not have the ſame vaniſhing line; for the ſame vaniſhing plane can, in ſuch caſes, be parallel only to one of them.

COROLLARY II.

All parallel original planes have the ſame vertical line, vertical plane, and parallel of the eye; for there can be but one line drawn through the center of the picture, perpendicular to the ſame vaniſhing line, nor but one line through the point of ſight, parallel to the ſaid vaniſhing line; neither can two different vertical planes paſs through the ſame point of ſight and ſame vertical line.

THEOREM XII.

If two original planes cut each other in a line parallel to the picture, their vaniſhing lines will be parallel to each other, as alſo their interſecting and directing lines, if neither of theſe laſt coincide.

SAME FIGURE.

Let the planes G P, Y G, be two original planes interſecting each other in the line B G, parallel to the picture S L, and let the planes M F, V H, paſs through the point of ſight E, parallel to the ſaid original planes, and then will the vaniſhing lines H L, F N, be parallel to each other, as alſo the directing lines K P, Q Y, and interſecting lines W X, S I.

DEMONSTRATION.

Let the plane ABG paſs through the line BG parallel to the picture and directing plane K M, then will the ſections B G, X W, Y Q, made by the plane YG, with the three planes ABG, S L, K M, be parallel to each other, (theorem XIV. part I.) and for the ſame reaſon the ſections B G, R T, K P, made by the plane G P, interſecting the ſame three planes, will alſo be parallel one to another, conſequently X W, R T, the interſecting lines of the two given original planes, will alſo be parallel, and their directing lines Q Y, K P, will

alſo be parallel. (theorem I. part II.) Now the vaniſhing lines H L, F N, of thoſe planes are parallel to their interſecting lines, by the aforeſaid theorem, and the original planes, not being themſelves parallel, they can not have the ſame vaniſhing line; (cor. I. to theorem VI. part II.) therefore the vaniſhing lines H L, F N, are diſtinct and parallel to each other. *Q. E. D.*

COROLLARY I.

All planes, whoſe vaniſhing lines are parallel, have the ſame vertical plane, vertical line, and parallel of the eye; for C n, which is at right angles to F N, is alſo perpendicular to H L, and M E, being parallel to F N, is alſo parallel to H L.

COROLLARY II.

If the interſection B G of the given planes were in the picture, it would then be their common interſecting line, and if it were in the directing plane, it would be their common directing line; but, in either caſe, their other lines would be parallel one to another.

THEOREM XIII.

If the vaniſhing lines of two original planes be parallel, their common interſection will be parallel to the picture.

SAME FIGURE.

Let the two original planes be the ſame as in the preceedent.

DEMONSTRATION.

Then (by cor. I. to the laſt theorem) the original planes, having the ſame vertical plane perpendicular to them both, it is alſo perpendicular to their common ſection B G; but the vertical plane is

perpendicular to the vaniſhing lines of the original planes; (theorem II. part II.) therefore B G, the common interſection of thoſe planes, is parallel to their vaniſhing lines H L and F N, and conſequently parallel to the picture. *Q. E. D.*

COROLLARY I.

If an original line B G be parallel to the picture, it will be parallel to the vaniſhing, interſecting and directing lines of all original planes which can paſs through the ſaid original line.

For all planes whatſoever, which paſs through the line B G, cut the plane A G in that ſame line; conſequently the vaniſhing, interſecting and directing lines of all ſuch planes are parallel to B G. (16. 11.)

COROLLARY II.

The original of the image of any line in an original plane, parallel to the vaniſhing line of that plane, is parallel to the picture.

For a line in an original plane, parallel to the picture, being parallel to the vaniſhing line of that plane, by the corollary above, its image muſt be parallel to the ſame line; (cor II. theorem IV. part II. and theorem XI. part I.) therefore a line, as B G, parallel to the picture, may be found in the original plane, which will produce the given image; but no two different lines in the ſame plane can produce the ſame image; therefore, if the given image be the image of a line in the original plane, it muſt be the image of B G, a line in that plane parallel to the picture.

THEOREM XIV.

If an original plane, being produced, paſs through the point of ſight, its vaniſhing and interſecting lines will coincide, and its directing line will be the ſame with the parallel of the eye.

ILLUSTRATION. Plate IV. Fig. 5.

Let B A Z R be the original plane, which, being produced to the line P L, passes through the eye at E, cutting the directing plane N Y in the line P L.

DEMONSTRATION.

Now, seeing that L B is the only plane which can pass through L P parallel to the original plane B A Z R, it follows that the original and vanishing plane coincide, and form one continued plane; therefore L P is both the directing line and the parallel of the eye, belonging to the said original plane, and x y, the intersection of the picture O P, with the original plane produced, is both its intersecting and vanishing line. *Q. E. D.*

COROLLARY I.

The image of an original plane, passing through the point of sight, is a line only, and in that line the images of all points, lines and figures, situated in the original plane, must be formed; wherefore the original plane, in this case, hath no depth. For a line, drawn from E, the point of sight, to any point in the plane BAZR, will cut the picture O P in some one of the points in the line x y; therefore the image of that point must be in that line x y.

COROLLARY II.

All original lines, situated in the plane B A Z R, have the same line P L for their director. For no line in the original plane can cut the directing plane N Y, but only in the line P L.

COROLLARY III.

Any line, drawn in the picture, may be the image of an original

line in a plane paſſing through the eye, and the line given in the picture; or it may be taken as the vaniſhing line of that plane, or as the image of its interſection with all other planes whatſoever which it interſects, as is evident from the above. (cor. I.)

COROLLARY IV.

The original of a line given in the picture can not be determined, unleſs two points in that line be known, one of which at leaſt muſt be an original point, but the other may be the vaniſhing point of the ſaid original line required.

For if the vaniſhing point alone be given, the direction only of the original line is thence determinable, as being parallel to the line producing that vaniſhing point, but the original line itſelf may be any line parallel to that ſaid line, and being in a plane paſſing through the eye and the line given in the picture; but when the direction of the original line and an original point in that line are known, the original line itſelf is then determined, by reaſon there can not be two different lines drawn through a given point parallel to the ſame line.

THEOREM XV.

The original of any figure in the picture, may be any object which is bounded by the ſame pyramid of rays indefinitely produced.

SAME FIGURE.

Let O P be the picture, E the point of ſight, A G B F the original figure, N L the directing plane, and a g b f the image of the original on the picture O P.

DEMONSTRATION.

Now g c may be the image of any original line in the plane G E C,

terminated by the lines E G, E C, indefinitely produced, and b c may be the image of any original line in the plane C E B, bounded by the lines E C, E B; in like manner produced (cor. III.) to the preceedent, it is the ſame with the other ſides of the given figure; therefore a g b f may be the image of any figure whatſoever, bounded by the ſame pyramid of rays E F B G A, indefinitely produced.

Q. E. D.

COROLLARY.

If the image of any plane figure be given in the picture, its original can not be determined, unleſs three points in that figure be known, one of which at leaſt muſt be an original point, but the other two may be vaniſhing points.

For the two given vaniſhing points will determine the vaniſhing line of the plane containing the original figure; (cor. III. to theorem IX. part II.) conſequently the direction of that plane is known, ſeeing it muſt be parallel to the vaniſhing plane which produces that vaniſhing line; (def. VIII. part II.) but, notwithſtanding this, the original plane is yet undetermined, for it may be any plane parallel to that vaniſhing plane; (theorem XI. part II.) whence ſome one point in the original plane is neceſſary to be known, in order to aſcertain that plane; which point being given, the original plane will be thereby determined, for two different planes can not paſs through the ſame point parallel to the ſame vaniſhing plane. Now the original plane, containing the original figure, being thus found, the original of the given image on the picture is truly aſcertained.

THEOREM XVI.

Any line in the picture, parallel to the vaniſhing line of an original plane, if it be the image of an original line, muſt be either the image

of a line parallel to the picture, or of one whose directing point is somewhere in the parallel of the eye belonging to that plane.

DEMONSTRATION.

For, if the original line be parallel to its image, it must also be parallel to the picture; but, if it be not parallel to its image, that image must then be parallel to the director of the original line; that director, being therefore parallel to the proposed vanishing line, (theorem XI. part I.) it must be the parallel of the eye belonging to that vanishing line, (theorem I. part II.) seeing there can not be drawn two different lines through the point of sight parallel to the same vanishing line. *Q. E. D.*

COROLLARY I.

Any line in the picture, parallel to the vertical line of an original plane, if it be the image of an original line, must be either the image of a line parallel to the picture, or of one, whose directing point is in the director of that plane. For, if the original line be not parallel to the picture, its image must be parallel to its director; which director, being therefore parallel to the proposed vertical line, it must be the director belonging to the original plane.

COROLLARY II.

Any two parallel lines in the picture, if they be the images of any two original lines, they must be either the images of lines parallel to each other and to the picture, or of such original lines as have the same director. For, if the original lines be parallel to their images, they must have the same director, by reason there can be but one director drawn parallel to the given images.

THEOREM XVII.

If two original planes cut each other in a line not parallel to the picture, their vanishing, intersecting and directing lines will also cut each other; and the intersection of the vanishing lines will be the vanishing point, the intersection of the intersecting lines will be the intersecting point, and the intersection of the directing lines will be the directing point of the common intersection of those planes.

ILLUSTRATION. Plate VII. Fig. 8.

Bring Q, in the plane K C, to coincide with Q in the plane B M, bring D, in the plane N D, to coincide with D in the plane B M, and make T, in the plane D V, to coincide with T in the plane B M, and make the plane L R pass through both the planes D V, K C, in the lines V T, C Q; bring D, in the plane D W, to coincide with D in the plane B M, and pass the plane W H through the plane L R, in the line H L.

Now, let the planes K C, B M, be two original planes, cutting each other in the line K Q, which is not parallel to the picture L R, let O be now the point of sight, Z B the directing plane, W H the vanishing plane of B M, and D V that of the plane K C; then are the lines H L, V T, the vanishing, T R, C Q, the intersecting, and P B, P N, the directing lines of the said original planes; I say, those several lines will cut each other, and the rest be as the theorem declares.

DEMONSTRATION.

Because the line K Q is not parallel to the picture, it must have a vanishing, intersecting and directing point. Now K Q, being a line common to both the original planes, its vanishing point V must be in both their vanishing lines; (theorem IX. part II.) whence H L, the vanishing line of the original plane B M, must cross S T,

the vanishing line of the other original plane K C, in the point V, seeing those lines can not coincide, by reason the proposed planes are not parallel to each other; (cor. I. theorem XI. part II.) therefore V is their common vanishing point.

For the same reasons the intersecting lines T R, C Q, and the directing lines P B, P N, or P Z, must cut each other, as here at Q and P, those lines being parallel to their respective vanishing lines; therefore Q is the intersecting, and P the directing point of the said original planes. *Q. E. D.*

COROLLARY I.

Hence it follows, that, if two original planes be both perpendicular to the picture, their vanishing lines will make, with each other, an angle equal to the angle of inclination, which the original planes have one to the other; and their common intersection will be in the center of the picture, which is also the common center of those said vanishing lines.

SAME FIGURE.

Raise up the plane D V, and make the plane L R pass through the same in the line V T, bring D, in the plane D W, to coincide with D in the plane B M, and pass the plane W H through the plane L R, in the line H L; make the line B W, in the plane D V, pass through and coincide with the line B W in the plane D W, and revolve the plane E X about the line E I, so as to pass through the plane L R, in the line X C.

Now, let the planes B M, D V, be the two original planes, perpendicular to the picture L R, then will the intersecting lines T R, V T, of those planes with the picture, determine their angle of inclination, which is, in this case, a right angle, they being here perpendicular to each other, as well as to the picture; and, as the in-

terſecting lines determine that angle, the vaniſhing lines H L, X C, or X Q, muſt alſo do the ſame, (10. 11.) and the line G T, which is the common ſection of the original planes, being alſo perpendicular to the picture, (19. 11.) its vaniſhing point is in C, which is the center of the picture; (def. V. part II.) therefore C is alſo the common center of the vaniſhing lines of the propoſed planes.

COROLLARY II.

Hence it alſo follows, that when two original planes D V, B M, are perpendicular to each other, as well as to the picture, their vaniſhing lines will be at right angles to each other, and the vaniſhing line of either plane will be the vertical line of the other plane; for the vaniſhing lines H L, X Q, are perpendicular to each other, becauſe they make with each other the ſame angle with that of the propoſed planes, by the preceedent corollary; and the vertical line of every plane, paſſing through the center of the picture, is perpendicular to the vaniſhing line of that plane; (theorem II. part II.) wherefore the vaniſhing lines H L, X Q, paſſing through C, muſt be reciprocally the vertical line one of the other.

COROLLARY III.

If two original planes D V, B M, be perpendicular to each other, and only one of them, ſuppoſe B M, perpendicular to the picture, their vaniſhing lines will ſtill be at right angles to each other, and the vaniſhing line H L, of that plane B M, which is perpendicular to the picture, will be the vertical line of the other plane D V, which is not perpendicular to the picture, but the vaniſhing line of the laſt-mentioned plane D V will not be the vertical line of the firſt-mentioned plane B M, but only parallel thereto; for the plane B M, being perpendicular to the picture, all lines, perpendicular to that

plane, are parallel to its vertical line X Q, and alſo to the picture; and ſeeing the planes B M, D V, are perpendicular to each other, a line may be drawn, in the plane B M, perpendicular to the plane D V, (38. 11.) and conſequently parallel to H L, and alſo to the picture L R; that line being therefore parallel to the vaniſhing lines of all planes which paſs through it, (cor. I. theorem XIII. part II.) it muſt be alſo parallel to H L, the vaniſhing line of the plane B M; (theorem XI. part I.) that vaniſhing line is therefore parallel to the vertical line of the plane D V, and conſequently perpendicular to the vaniſhing line thereof. Now the line H L, paſſing through C, the center of the picture, at right angles to the vaniſhing line of the plane D V, is the vertical line of that plane; but, as the plane D V is, by hypotheſis, not perpendicular to the picture, its vaniſhing line can not paſs through C, and conſequently H L can not, in this caſe, be the vertical line of the plane D V, but parallel thereto only.

THEOREM XVIII.

If an original line be given in a plane, parallel to the picture, it will be in the ſame proportion to its image on the picture, as the diſtance between the eye and original plane; is to the diſtance between the eye and the picture.

PLATE IV. Fig. 5.

Let O P be the picture, c its center, E the point of ſight, A B a line given in the plane S T parallel to the picture, E c the axis of the eye produced till it meets the original plane in C; then, as the original line A B is to its image a b, ſo is E C, the diſtance of the eye from the original plane, to E c, the diſtance of the eye from the picture.

DEMONSTRATION.

Becauſe A B is in a plane parallel to the picture, it is itſelf parallel to the picture, and alſo to its image a b; (cor. II. theorem IV. part II.) wherefore the triangles E A B, E a b, are ſimilar; and, for the ſame reaſon, the triangles E C B, E c b, are alſo ſimilar; whence, in the ſimilar triangles E A B, E a b, we have

A B : a b :: E B : E b,

and, in the ſimilar triangles E C B, E c b, it is

E B : E b :: E C : E c, conſequently

A B : a b :: E C : E c. *Q. E. D.*

COROLLARY I.

If the original line A B be any wiſe divided into ſeveral parts, the images of thoſe parts will have the ſame ratio one to the other, as the original parts bear one to another: becauſe each original part is to its image as the diſtance between the point of ſight, and original plane, is to the diſtance of the eye from the picture, which two diſtances remain invariable; and, for the ſame reaſons, the images of the ſides of any figure A B G F, in a plane parallel to the picture, have the ſame proportion to each other, as the correſponding ſides of the original figure have one to the other.

COROLLARY II.

The image of any figure, in a plane parallel to the picture, is ſimilar to its original; becauſe the angles, made by the ſides of the image, are equal, and its ſides proportional to the correſponding angles and ſides of the original figure, by cor. IV. theorem IV. part II. and the preceedent.

COROLLARY. III.

The image of a given line, in a plane parallel to the picture, will be of the same length, in whatever point of the directing plane the eye be fixed; because the distance between the eye, the picture, and the original plane, undergoes no alteration, let the point of sight be taken in the directing plane where it will; therefore the proportion of the image to its original remains the same.

COROLLARY IV.

If the picture and original plane are both on the same side of the eye, the greater the distance is between the eye and original plane, the nearer does the image of any given line in that plane approach to equality with its original.

ILLUSTRATION. Plate V. Fig. 6.

Raise up the planes P Z, D K, and make the planes M N, S Q, pass through the plane D K; bring R to coincide with R, and make the plane K A pass through the planes M N, S Q, P Z.

Now let A D be a given line in the original plane P Z, and S Q the picture, a d its image when seen by the eye at E; suppose now the eye to be removed farther from the original plane, as at K, then, drawing the lines D K, A K, the line F G will be the image of A D, when viewed by the eye placed in K; whence, seeing the original line A D is to its image seen at E, as E L is to E C, by the precedent theorem, the same line A D will be to its image F G, seen at K, as K L is to K C; but K E, being greater with respect to E C, than it is with respect to E L, (8. 5.) K E + E C = K C, will be greater in respect of K E + E L = K L, than E C is to E L; and consequently F G, the image of A D, when seen from K, will be greater in proportion to A D, then it will be when seen from E.

Now, seeing the image of A D is always less than its original when thus situated with respect to the eye and the picture, and, as the image becomes greater in proportion as the distance of the eye is increased; it must therefore approach nearer to an equality with the original line first given, in proportion to the remoteness of the point of sight. *Q. E. D.*

SCHOLIUM.

It having been shewn, in cor. I. to theorem V. part II. that the directing point of an original line can have no image on the picture, it follows, that all points whatever of an original line, which can be represented on the picture, must be at some distance from the original plane. Now the line on the picture, bounded by the image of any point in an original line, and the vanishing point of the said line, is called the INDEFINITE IMAGE of that said original line. And the image of any determinate part thereof, bounded by any two points given therein, is the PERSPECTIVE IMAGE of that determinate line so bounded. And the remainder of the indefinite image is called the COMPLEMENT OF THE IMAGE. And that part of the original line, contained between its directing point and that point of the original line which is nearest the directing point, is called the COMPLEMENT OF THE ORIGINAL LINE.

PLATE VIII. Fig. 9.

Raise up the plane G E, and make the picture S H pass through the same, make D coincide with D, and the plane M H pass through the picture S H, in the line H L.

Now, let A G be an original line, C its vanishing point, A the nearest point thereof to its directing point D; then is a C the indefinite image of A G. If G had been the nearest point of the line re-

presented on the picture, g C would then have been the indefinite image of that line. If both the points A and G be represented on the picture, as here in the points a and g, the line a g is the perspective image of that determinate original line A G. Now a g being the perspective image, g C is the complement of that image, and A D is the complement of the original line A G.

THEOREM XIX.

If an original line be divided into any two parts, the rectangle contained between the extremes of the indefinite image thereof, will be to the rectangle contained between the mean part thereof and the whole indefinite image, as that part of the original line, which is nearest to the directing point, is to the other more distant part thereof.

SAME FIGURE.

Let A G be the original line, any how divided in the point K, into the two parts A K, K G, and the indefinite image thereof, a C, into the three parts a k, k g, g C; I say, the rectangle under the extremes g C, and a k, is to the rectangle under the mean part k g and the whole indefinite image a C, as A K, the nearer part of the original line A G, is to K G, the other part thereof.

DEMONSTRATION.

Through a, the image of the point A, draw the line T V parallel to the original line A G, cutting E G and E K in the points T and X, then are the triangles E C k, k a X, similar one to the other; wherefore $aX : EC :: ak : kg + gC$,

and the triangles E C g, g a T, are also similar; whence we have

$$EC : aX + XT :: gC : ak + kg,$$

and, by multiplying these two proportions together, the product will be

$$aX \times EC : EC \times aX + EC \times XT :: ak \times gC : kg \times ak + kg \times kg + gC \times ak + gC \times kg.$$

But, by (1. 2.) $aC \times kg = kg \times ak + kg \times kg + gC \times kg$;

whence $aX \times EC : EC \times aX + EC \times XT :: ak \times gC : gC \times ak + aC \times kg.$

Now, subtracting the antecedents from the consequents, it is

$$aX \times EC : EC \times XT :: ak \times gC : aC \times kg;$$

that is, $aX : XT :: ak \times gC : aC \times kg$,

but, seeing that AG and TV are parallel to each other by construction, it will be $aX : XT :: AK : KG$;

therefore $ak \times gC : aC \times kg :: AK : KG$, *Q. E. D.*

SCHOLIUM.

Although the line A G is here supposed wholly beyond the picture, the proposition is nevertheless true universally, wherever the point A be taken in the line A G, so long as the line C F is parallel to E D; for any line whatever, that is parallel to C F or E D, cutting the lines E C, E A, E K, and E G produced if necessary, will be thereby divided into parts, bearing the same ratio to each other, which the correspondent parts a k, k g, g C, of the line C F, have one to another.

COROLLARY I.

The image a k of A K, the nearer part of the original line A G, is to k g the image of the farthermost part K G, as the rectangle under A K the nearer part, and the whole line G D, or line G A, produced to its directing point D, is to the rectangle under K G and D A, the extremes of the whole line G D.

For, through g draw g Q parallel to A G, cutting the lines E A, E K, in the points R and P; then, if a g be considered as an original line, C as its directing point, E C as its director, and Q g as its indefinite image, it follows, from the preceedent theorem, that

$$RP \times Qg : Pg \times QR :: ak : kg,$$

and because Q g and A G are parallel,

$$Qg : QR :: DG : DA,$$

also $RP : Pg :: AK : KG$;

therefore $RP \times Qg : Pg \times QR :: DG \times AK : DA \times KG$;

consequently $ak : kg :: DG \times AK : DA \times KG$.

COROLLARY II.

If the parts A K, K G, of the original line, be equal to each other, then a k, the image of the nearer part, will be to k g the image of the remoter part, as the indefinite image a C is to its complement g C.

For, by the preceedent theorem, $ak \times gC : kg \times aC :: AK : KG$.

Now, since by hypothesis, $AK = KG$,

$$\text{it will be } ak \times gC = kg \times aC;$$

whence arises this analogy, $ak : kg :: aC : gC$; wherefore the line A C, in this case, is divided into harmonical proportion, by the points k and g.

COROLLARY III.

The hypothesis being the same as in the preceedent corollary, it will be

$$ak : kg :: DG : DA.$$

For, in the similar triangles E V T, E C g, we have

$$EV = aC : gC :: VT : EC = Va;$$

but, by the last corollary, $ak : kg :: aC . gC$.

P

Now, seeing V T and D G are parallel by hypothesis,

$VT : Va :: DG : DA$;

therefore $ak : kg :: DG : DA$.

COROLLARY IV.

If the images a k, k g be equal, then A K, the nearer part of the original line is to K G, the farther part thereof, as D A, the complement of the original line, is to D G, the whole line G A, when produced to it's directing point D.

For, by the above corollary I. $ak : kg :: AK \times DG : KG \times DA$.

Now, if $ak = kg$, then is $AK \times DG = KG \times DA$;

whence arises this analogy, $AK : KG :: DA : DG$.

COROLLARY V.

The hypothesis being the same as in the preceedent corollary, it will be $AK : KG :: gC : aC$;

for the triangles E V T, E C g, being similar, it is

$EC = Va : VT :: gC : aC$;

but $Va : VT :: DA : DG$.

And, by the preceedent corollary,

$DA : DG :: AK : KG$;

consequently $AK : KG :: gC : aC$.

C O N C L U S I O N.

HAVING thus ſhewn the ſeveral poſitions of the planes, vaniſhing lines, points, &c. neceſſary for conveying to learners a general idea of the nature of perſpective repreſentations, when formed ſtereographically on the plane of the picture; we now proceed to conſider them in another view, ſuch as is required in the practice of drawing the perſpective repreſentations of any given original objects, or for determining the original objects from having their reſpective repreſentations on the picture given; and in this we are to conſider them each as being bounded by its own proper dimenſions, and ſo placed with relation to each other, as to coincide together, and form one continued plane.

P L A T E IX. Fig. 10.

Let there now be given the center of the picture, the height of the eye, the diſtance of the eye from the picture, and the poſition of the original plane with reſpect to the picture. Thus, ſuppoſe P M to repreſent the original plane, L M the picture conſidered as being perpendicular thereto, L X the vaniſhing plane, E the point of ſight; then, upon the paper or cloth, &c. intended for the picture which is to contain the required repreſentations, draw at pleaſure the line M N, and parallel thereto, at a diſtance equal to the given height of the eye, draw the line H L, and perpendicular thereto, through the point of ſight E, draw the line E r, interſecting the lines H L, M N, in the points C and r, and through E, parallel to H L, draw the line Z X; then is M N the interſecting line, H L the vaniſhing line, Z X the parallel of the eye, C r the vertical line, E C the diſtance of the eye from the picture, C the center of the picture, and alſo that of the vaniſhing line H L, they both coinciding together in this particular poſition of the picture. This preparation being made,

the ſeveral planes and lines here mentioned are in the proper ſituations for the practice of drawing the repreſentations required to be determined by the help of thoſe ſeveral planes, &c. As to the particular dimenſions proper to be aſſigned to thoſe planes, we ſhall treat thereof in the annotations which are hereto annexed.

PROBLEM I.

To find the indefinite image of an original line, not parallel to the interſecting line, by help of the vaniſhing and interſecting points thereof.

CONSTRUCTION.

Let the line N I be an original line, meeting the interſecting line M N in the point I; from E draw the line E V parallel to N I, cutting the vaniſhing line H L in the point V, draw the line I V, and it will be the indefinite image required.

DEMONSTRATION.

The original and vaniſhing planes being parallel to each other, (def. VIII. part II.) and the line E V parallel to N I, by conſtruction, the point V is the vaniſhing point of the line N I; (def. XIX.) now the interſecting line M N being common both the plane of the picture and the original plane, I is therefore the interſecting point of the given original line N I; conſequently I V is the indefinite image thereof, by the ſcholium in page 50.

SCHOLIUM.

The line E V muſt be ſo drawn, that the vaniſhing point of the given line may be on the ſame ſide of the vertical line, to which the inclination of the original line tends towards the interſecting line; thus N I, being the original line, its vaniſhing point V falls between

C and H; but if D M be the original line, then its vaniſhing point V falls between C and L; and if the original line be perpendicular to the interſecting line, as the lines A N, B D, &c. then will the vaniſhing point fall in C, the center of the picture.

PROBLEM II.

The indefinite image of a line being given; thence to find the original, by help of the vaniſhing plane and interſecting point thereof.

CONSTRUCTION.

Let I V be the indefinite image given; produce the ſame both ways when neceſſary, ſo as to meet its vaniſhing and interſecting points V and I, and from I draw a line, as I N parallel to E V.

DEMONSTRATION.

Then the line I N, being drawn through the interſecting point I, parallel to the line E V, drawn in the vaniſhing plane through the point of ſight, by conſtruction; it is therefore the original line required. (def. XIX.) *Q. E. D.*

PROBLEM III.

The common vaniſhing point of parallel lines in the original plane being given; thence to find the vaniſhing point of all other lines in that plane, which make a given angle with the lines firſt propoſed.

CONSTRUCTION.

Let T O, S U, be two given original lines parallel to each other, V their common vaniſhing point, and let S T, U O, be two other given lines, making any given angle S T O, or U O T, &c. with the former. From the given vaniſhing point V draw a line to the point of ſight E; then from E, towards that end of the vaniſhing line to which

Q

the lines tend, whoſe vaniſhing point is ſought; draw the line E y, making with V E an angle V E y, equal to the given angle, and the point y, in which the line ſo drawn cuts the vaniſhing line, is the other vaniſhing point required.

DEMONSTRATION.

By ſcholium, in page 50, the lines o V, u V, are the indefinite images of the given parallel lines T O, S U, and o y, t y, are thoſe of the lines U O, S T, and the angle V E y = T O U, by conſtruction; therefore V and y are the vaniſhing points of thoſe ſeveral lines. (theorem VI. part II.) *Q. E. D.*

COROLLARY I.

If the original lines propoſed, as A B, N D, be parallel to M N, the interſecting line of the original plane, then the vaniſhing point of ſuch lines, as AD, B N, which make a given angle with the propoſed original lines, may be found thus; through E draw E V, to cut the vaniſhing line in V, on the ſame ſide or ſides of C, to which the required originals are ſuppoſed to tend, making the angle X E V, or its equal E V y, equal to A B N, or D N B, the angle propoſed; then will V be the vaniſhing point of all ſuch lines as B N, and v that of all ſuch lines as A D. For the line E V makes the ſame angle with the eye's parallel E X, as the originals, whoſe vaniſhing point is V, make with the interſecting line of the original plane, or any line parallel thereto. The ſame holds good with reſpect to the line E v, and the lines whoſe vaniſhing point is v.

COROLLARY II.

Hence it follows, that if the original lines propoſed incline ſo much to the interſecting line of the original plane, as to cauſe their

vaniſhing point to be out of reach; yet if the angle, which the original lines make with the interſecting line, be known, the vaniſhing point of lines, which make a given angle with the propoſed original lines, may be found in manner as above; for, drawing E V on that ſide of C, to which the original lines are ſuppoſed to tend, making the angle X E V equal to the angle of inclination which the original lines make with the interſecting line; draw another line E v, cutting the vaniſhing line in v, and making the angle V E v equal to the angle propoſed, and v will be the vaniſhing point deſired, for the ſame reaſons as above.

LEMMA.

The director of a line in an original plane makes the ſame angle with the eye's parallel and directing line of that plane, as the image of the given line makes with the vaniſhing and interſecting lines of that plane. Becauſe the director and the image of the original are parallel one to the other. (theorem VII. part II.)

COROLLARY I.

The director of an original line makes the ſame angle with the eye's director of that plane, which the image of the original line makes with the vertical line of that plane.

Becauſe the eye's director and vertical line are parallel, as is alſo the director of the original line to the image thereof.

COROLLARY II.

The indefinite images of all lines whatſoever, whoſe directing points are any where in the parallel of the eye, relating to an original plane, are parallel to the vaniſhing line of that plane.

For the images are parallel to their director, which being the pa-

rallel of the eye, in this caſe, they are therefore parallel to the vaniſhing line of the original plane.

COROLLARY III.

The indefinite images of all lines whatever, whoſe directing points fall any where in the eye's director, relating to an original plane, are parallel to the vertical line thereof, whether the propoſed lines be in or out of that plane.

For the director of the eye, being the director of the propoſed lines, it is therefore parallel to the vertical line.

COROLLARY IV.

If any two lines in an original plane cut the directing line of that plane in the ſame point, their images will be parallel.

For the original lines muſt then have the ſame director.

COROLLARY V.

If the images of any two original lines in the ſame plane be parallel, the original lines, if they be not parallel to the picture, muſt have the ſame directing point.

For there can be drawn but one director parallel to both the given images, and that director can cut the directing line of the original plane only in one point, which point is therefore the common directing point of the propoſed original lines.

SCHOLIUM.

It is eaſy to reverſe this problem, that is, from a directing point given, thence to find another directing point, ſuch that the images of all original lines, which have thoſe points for their directing points, may make in the picture an angle equal to any angle pro-

posed, only by using the directing plane and directors in like manner as the vanishing plane and the lines drawn to the vanishing line from the point of sight parallel to the given original lines, were used in the above, regard being had to the position of the directing point sought, which must fall on the contrary side of the given directing point to that, whereto the proposed images are intended to incline, the demonstration whereof is deducible from the preceedent lemma and its corollaries.

N. B. The angles determined by this problem, when neither of the original lines are supposed parallel to the intersecting line, are those comprehended between the two vanishing points, at the two intersecting points of the images, as V m v, or M m I, or the corresponding angles of the originals, and not the angles which the originals, or their images make side ways, as V m M, or v m I.

DEFINITION.

The angles V m v, M m I, or any others in a like position with their corresponding originals, are here called INTERNAL ANGLES to distinguish them from the angles V m M, v m I, which are called external angles.

PROBLEM IV.

A vanishing point being given; thence to find two other vanishing points, such that all lines, drawn in the picture from those three points, on the same side of the vanishing line, may, by their mutual intersections, form triangles, whose originals shall be similar to an original triangle given.

PLATE X. Fig. 11.

Let the same things be supposed as before, and let V be the vanishing point given.

CONSTRUCTION.

Draw the line E V, and from E take any part thereof, as E W; upon the line E W make a triangle E W Z similar to the original triangle proposed, having either of its angular points in E; then produce E Z till it cut the vanishing line in some point, as H, from E draw another line, as E K, parallel to W Z, cutting the said vanishing line in K; then are H and K the vanishing points required. Let there now be drawn, from the points H, V and K, any lines, as H m, H p, H s; V n, V m, V p; K k, K n, K m; making by their mutual intersections any triangles, a m p, a b k, a k n, a n m, &c. I say, the originals of all those several triangles are similar to the original triangle proposed.

DEMONSTRATION.

Because the originals of a p, a m, m p, whose vanishing points are H, V and K, are respectively parallel to the lines E H, E V, E K, drawn from the point of sight to those vanishing points, (def. XIX. part II.) and E K is parallel to W Z by construction; therefore the original of m p is also parallel to W Z, (theorem XI. part I.) for the same reasons, the originals of the three sides a m, a p, m p, of the triangle a m p, being respectively parallel to the three sides E W, E Z, W Z, of the triangle E W Z, and their corresponding angles being equal; (theorem XII. part I.) therefore the original of the triangle a m p is similar to the triangle E W Z, but the triangle E W Z is similar to the original triangle given, by construction; whence the proposition is manifest. The same may, in like manner, be proved of each of the other triangles a b k, a k n, &c. *Q. E. D.*

SCHOLIUM.

If it happens, in constructing the triangle E W Z, similar to the

original triangle given, that the line W Z be parallel to the vanishing line H L, then the original triangle will have but two vanishing points, from which drawing any two lines so as to intersect each other, and these being again intersected by any line drawn parallel to the intersecting line M N, a triangle will thereby be formed, whose original is similar to the triangle proposed.

Suppose now, in the same figure, S T to be parallel to H L or X Y, then a line drawn through E, parallel to S T, will coincide with X Y; whence the side of the original triangle, corresponding to S T, can have no vanishing point in this case; (theorem IV. part II.) it will therefore be parallel to the intersecting line M N, (cor. I. theorem XIII. part II.) to which its image is also parallel; (cor. II. theorem IV. part II.) wherefore any two lines H R, K D, being drawn from the vanishing points H and K, any where cutting each other, as in m, any lines g f, n p, drawn parallel to the intersecting line M N, or even the intersecting line itself, will, by their intersections with the lines H R, K D, form triangles m p n, m g f, D m R, &c. whose originals will be similar to the original triangle given.

COROLLARY

Hence, if three vanishing points, or two only, in such cases as the above, of a triangle be given, the species of the original triangle may be thence determined.

Thus, suppose H, V and K to be given, draw the lines E H, E V, E K; then, through either two of those adjacent lines, draw a line parallel to the third, and it will determine the species of the original triangle; for, drawing W Z through E H, and E V, and the line W Z parallel to E K, the triangle E W Z will be similar to the original triangle, from what has been premised, or a line W Q, drawn through E V, and E K, parallel to E H, determines E W Q the species of the

original triangle; for the triangles E W Z, E W Q, are ſo evidently ſimilar, as not to need a demonſtration thereof; or if H and V be the only two vaniſhing points of the original triangle, it is evident, that by drawing S T parallel to X Y, it will determine the ſame by the triangle E S T, which is alſo ſimilar to the triangle E H K.

SCHOLIUM.

In this problem it is limited, that the lines, drawn from the given vaniſhing points of the given original triangle, be all on the ſame ſide of the vaniſhing line, in order that the triangle, formed by the interſections of thoſe lines, may repreſent a triangle ſimilar to the original; for if thoſe interſections fall ſome on one ſide, and ſome on the other, of the vaniſhing line, the original of the triangle ſo formed will be two diſtinct and ſeparate indeterminate figures.

The reverſe of this problem, that is, from a directing point given, to find two other directing points, ſuch that the images of all original lines, drawn from thoſe three directing points, may, by their mutual interſections, form on the picture, triangles ſimilar to a given triangle, may be eaſily performed, by making uſe of the directing plane, and the directors of the original lines, in the ſame manner as was ſhewn, of the vaniſhing plane and the lines drawn from the eye to the vaniſhing points of the given original lines, as was obſerved in the ſcholium page 60, which is ſo evident, that it needs no farther explanation.

PROBLEM V.

To find the image of any given point in an original plane.

This is done by finding the indefinite images of any two lines in

the original plane which paſs through the given point, for the interſection of thoſe images is the image of the point required. (theorem VIII. part II.)

EXAMPLE I. PLATE IX. Fig. 10.

For, let z be the point given, D M, N I, two lines in the original plane, interſecting each other in that point, M v, I V, their indefinite images on the picture, interſecting each other in the point m; then is that point m the image or perſpective repreſentation of the point z, which was to be found.

EXAMPLE II.

To find the perſpective repreſentation of B D, a line perpendicular to the interſecting line M N, or bottom of the picture; draw from one of the extremes, as D, a line D M, any how cutting the interſecting line, as here at M; through E, parallel to D M, draw E v, cutting the vaniſhing line in v, the vaniſhing point of D M, through B, the other end of the given line, draw N I, and parallel thereto draw E V, continue B D till it meet the interſecting line as at i; then drawing M v, I V and i C, the intercepted line b d will be the perſpective repreſentation of B D required.

EXAMPLE III.

To find the perſpective repreſentation of A B, a line parallel to the interſecting line or bottom of the picture.

Draw B i, A x, each perpendicular to M N, and meeting the ſame in the points i and x; draw x C, i C, and the other lines as before; then is a b the repreſentation required.

EXAMPLE IV.

To find the repreſentation of a line, as NB, in an oblique poſition with the interſecting line.

Produce N B to I, and draw the ſeveral lines as before directed, then will the line b n be the repreſentation required.

SCHOLIUM.

Now, ſeeing that all plane right lined figures are bounded by lines, which are either perpendicular, parallel or oblique to the interſection line, the examples here given ſuffice for finding the repreſentations of any given rectilinear figures, the picture, original and vaniſhing planes being in the ſituation here deſcribed, we might now proceed to the finding any other repreſentations, or to the finding of the originals themſelves, from having their repreſentations given on the picture, in every ſituation that can be given to the picture, with reſpect to the original plane; but that would be foreign to the deſign of this little tract, which is not intended to teach the art of drawing perſpective repreſentations in its full extent, but only to remove the difficulties which lay in the way of learners at their firſt entrance on this ſtudy, through their not being able readily to conceive, with ſufficient clearneſs, the actual poſitions of the ſeveral planes and lines neceſſary to be duly diſtinguiſhed and applied in ſtudying the rudiments of this art; wherefore, hoping this ſhort Eſſay will be of ſome aſſiſtance to learners in this reſpect, ſhall now endeavour to ſhew them, by a kind of ocular proof, the exact agreement which ſubſiſts between the practical conſtructions which are uſed in this art, and what we have previouſly advanced under a ſtereographical conſideration. To effect which, let us now ſuppoſe the picture to be raiſed at right angles to the original plane, by bringing P to coincide with P, and Q with Q; alſo ſuppoſe the va-

nishing plane parallel to the original plane, by making Y coincide with Y, and W with W; then, supposing A B D N, G F K P R, O U S T, to be three original figures given in an original plane, and a b d n, g f k p r, o u s t, to be their respective representations on the picture, as investigated by the rules of practical perspective, when those several planes formed one continued plane: let now the silks be drawn through the point of sight, they representing so many rays of light issuing from the extremities of the given objects, and all uniting together in the eye or point of sight; then, by what was delivered in def. I. part II. it appears, that those several rays will, by their intersections with the picture, mark thereon the several points requisite for determining the required representations of the given original objects; and, seeing those rays intersect the picture in the very same points as were before found by practical construction, it is plain the result is the same in both; whence the perfect coincidence between the theory and practice of forming perspective representations appears manifest.

And the same may, in like manner, be shewn, let the position of the picture and the original plane be what it will; whether considered as parallel to each other, as obliquely situated one to the other, as inclining to, or as reclining from each other, under any given angle whatever, &c. which, it is hoped, will evidently appear from what has been delivered, and is contained in the following annotations; wherefore, having thus endeavoured to elucidate the doctrine of planes as applied to the primary operations used in the practice of perspective, which is the proposed boundary of this attempt, I now refer to Mr. Kirby's treatise afore-mentioned, and those masters whose province it is to exhibit the uses of those principles under the different modifications required in the practical applications thereof,

leaving to another opportunity the consideration of what may hereafter appear farther necessary for rendering this method of explaining the theory more extensive.

ANNOTATIONS,

OR

GENERAL COMMENT ON PART II.

I. OF MATHEMATICAL PROJECTIONS IN GENERAL.

MAthematical projections are of two kinds, viz. geometrical and ſtereographical; the former whereof ſhould be firſt underſtood, as being uſeful and neceſſary to the latter.

II. OF GEOMETRICAL PROJECTIONS.

Geometrical projections are conſtructed by drawing lines, parallel to each other, from the ſeveral points of the given objects, cutting the plane of projection either perpendicularly or obliquely, under any angle whatever. In this kind of projection, the place of the eye is not conſidered otherwiſe than by ſuppoſing it very remote, or at an infinite diſtance from the plane of projection; whence it can repreſent only two dimenſions at a time, as length and breadth without thickneſs, or length and thickneſs excluſive of breadth, &c. The projected images of objects differ according as they are ſituated with reſpect to the plane of projection: hence ariſe three caſes.

1. When the objects to be projected are in planes parallel to that of projection. 2. When perpendicular thereto. 3. When inclining to the ſame.

In the first of these cases, the images are exactly equal and similar to their originals; in the second, they become straight lines; and in the third, are neither equal nor similar.

CASE I. ILLUSTRATION. Plate I. Fig. 1.

Make F coincide with F, and the planes BM, SH, to pass through each other in the line HL. Now, let ABED represent an object to be projected on a plane SH parallel thereto. Draw AT, DR, BH, EC, parallel to each other, and perpendicular to the plane of projection SH, meeting the same in the points T, R, H, C; then, drawing the lines RC, CH, HT, RT, the figure HCRT is the geometrical projection of ABED, and exactly equal and similar thereto.

DEMONSTRATION.

The planes BD, RH, are parallel to each other by hypothesis, BH parallel to EC, and both perpendicular to the plane RH by construction; HC is therefore equal and parallel to BE. In like manner it may be proved, that the lines RC, RT, TH, are respectively equal and parallel to ED, DA, AB; and all the angles of the given figure being equal to the correspondent ones in the projection, therefore is RCHT equal and similar to ABED. Or if the projecting lines are oblique to the plane of projection, thus;

Let Ao, DV, BP, Ed, be drawn from the same points A, D, B, E, any how inclining to the plane RH, and meeting it in the points o, V, P, d, it may be proved as above, that the figure VoPd is also equal and similar to ABED. *Q. E. D.*

COROLLARY

Hence it appears, that the projection will always be the same,

whether the plane of projection be before, behind, above, below, nearer to, or farther from the original plane, provided the projecting lines are parallel to each other.

CASE II. SAME FIGURE.

Suppose now E f p q, in the plane B M, to be the object to be projected on the plane R H, perpendicular thereto; then, drawing f r, E C, to meet R H perpendicularly, and they will give the line C r, for the projection of E f p q, for all the lines that can be drawn from any points in that figure, to meet the said plane perpendicularly, will fall in the common section C r; therefore the projection, in this case, is a straight line only.

CASE III. SAME FIGURE.

Make the plane S H pass through the plane D Y; let now the plane D Z be the plane of projection, D Y an original plane containing a line a o, inclining to the plane D Z.

Draw o R, a e, perpendicular to D Z, and they determine the projection e R of the given line a o.

SCHOLIUM.

In this case, when the projecting lines are perpendicular to the plane of projection, each line in the given figure will be, to its image, as radius is to the co-sine of the angle of inclination. For, from the extreme o, draw o b parallel to e R, and it will also be equal thereto; whence o b a, being a right-angled triangle, o b = e R, is the co-sine of the angle of inclination; therefore it is a o : o b = e R :: Radius : the co-sine of the angle of inclination.

OF THE DIFFERENT WAYS OF APPLYING GEOMETRICAL PROJECTIONS.

There are ſeveral ways of applying this kind of projection, in all which the projecting lines are generally conſidered, as being perpendicular to the plane of projection; whence it is diſtinguiſhed by different names, ſuch as Ichnography or plan, Orthography or elevation, Section, Profile, &c.

Ichnography or plan, when it deſcribes the perpendicular ſeat or place which objects have on the ground, not regarding their heights above it. Thus the plan of a town, fort, or any building, is the geometrical deſcription of the perpendicular poſitions which their ſeveral parts have with reſpect to the ground, that being conſidered as the plane of projection.

Orthography or elevation, when it repreſents the plane or ſurface of objects, according to their breadth and heighth above the ground, excluding the depth or ſpace which they occupy thereon. Thus the elevation of a building is the geometrical deſcription of ſome one front or ſide thereof, deſcribed upon the plane of projection, as to the height and breadth thereof. And if the building be ſuppoſed to be cut by a plane paſſing through the ſame, and the ſeveral parts thereof be deſcribed upon that plane, according to the true meaſures which the parts have through which that plane paſſes; this is called a section of that building.

Profile, when it is applied to works of fortification, for deſcribing the ſeveral heights thereof, as they would appear upon a plane paſſing through them perpendicular to the ground.

Chart or map, when uſed geographically, to deſcribe the form of any part of the earth, and the poſitions of places ſituated upon the ſame, without conſidering the ſphericity thereof, but looking upon the part ſo deſcribed as being a plane.

In all these applications, the plane of projection is conceived as parallel to the plane of the original or thing described; whence the projection should, in strictness, be equal and similar to its original, but, in the practical uses hereof, it is similar only; for it is chiefly used to represent or describe large things in a smaller compass than they really possess, so as to be in any assigned proportion to the original, at the same time preserving, as much as possible, the true form and position of the several parts thereof.

This kind of projection is also useful to astronomical purposes, in projecting the sphere and its several circles in plano, in which those circles are considered as being in planes, whereof some are parallel, some inclining, and others perpendicular to the plane of projection; and they are accordingly represented on the given plane, by circles in the first case, ellipses in the second, and by straight lines in the third.

OF STEREOGRAPHICAL PROJECTION.

Stereographical projection is when the projecting lines, drawn from the original object to cut the plane of projection, are not parallel to each other, but converge together, and all meet in one point.

Objects are here considered as seen from one certain point, called the point of sight; whence this kind of projection can represent at once, all the three dimensions, length, breadth and thickness; that is, as it were, the solidity of objects. Here also arise three cases or varieties proceeding from the different positions which the point of sight, plane of projection, and the original plane may have with respect to each other.

1. When the plane of projection is between the point of sight and the object.

U

2. When the object is between that point and the plane of projection.

3. When the said point is between the object and the plane of projection.

The first of these cases is called perspective, and here the image is always less, in the second it is always greater, and in the third it may be either less, equal, or greater than the given object. But it is not true, that the PARTS OF A MAGNITUDE MAY BE LESS, EQUAL TO, OR GREATER THAN THE MAGNITUDE ITSELF *. It is only the representation of magnitude that is here described.

ILLUSTRATION. Plate V. Fig. 6.

Raise up the plane K D, and make the planes S Q and M N pass through the same; raise up the plane P Z, bring R to coincide with R, and pass the plane K A through the planes M N, S Q and P Z.

Now, in the first case, let E be the point of sight, the plane S Q, the picture or plane of projection parallel thereto, and P Z the original plane containing an original line A D. Then, drawing the lines E A, E D, they will intersect the picture or plane S Q, in the points a and d, and thereby determine the line a d for the perspective appearance of the original A D. Now it is evident, that a d must always be lesser than A D, and that if the plane S Q be situated nearer the plane P Z, every thing else being as before; the image a d will be greater and greater, as that plane approaches nearer to the plane P Z, and reverts into the original, by the coincidence of those two planes. On the other hand, if the plane S Q be supposed to approach, in like manner, the plane M N, the image decreases, and when those two planes coincide together, it will vanish into the point E.

* See page 27. Elements of Mathematics, &c. for the use of the Royal Academy at Woolwich, 1757.

In the ſecond caſe, let now the plane S Q be conſidered as the original plane, a d the original object, and the plane P Z to be the picture or plane of projection, and E the point of ſight, as before; then producing E a, E d, till they meet the plane of projection, as here at A and D, the line A D will be the image of a d, and greater than it, as is evident from the conſtruction itſelf, and becomes ſtill greater, as the plane S Q comes nearer to the plane P Z, and will become infinite, or rather be no image at all, when thoſe two planes coincide; for then the lines, which ſhould produce the image, will be parallel to each other, as in geometrical projections, therefore can not meet in a point to form an image, which ſtereographical projection requires them to do.

Laſtly, ſuppoſe B S to be the original object in the plane R W, and S Q the plane of projection parallel thereto, and E the point of ſight; then drawing through E the lines B E, S E, producing them till they meet the plane of projection, as here in the points d and a, and they will determine the line d a for the image of B S. Now, when K E = E C, it is evident that the image d a will be equal to the original B S, as in the poſition before us; but when K E is leſs than E C, the image d a will be greater than B S; but if E K be the greater, then the image will be leſſer than the original, and that when E C is nothing, or that the planes M N and S Q coincide, the image then vaniſhes into the point E; but if E K becomes nothing by the coincidence of K with E, the image ſhould then be infinitely great, and therefore can not be repreſented on the plane of projection.

COROLLARY.

Hence it appears, that, in all theſe three caſes above-mentioned, the point of ſight, plane of projection, and the original plane, muſt be at ſome diſtance from each other; for if either the plane of pro-

jection, or the original plane, coincides with the eye, no image can then be formed; and, if the plane of projection and original plane coincide, the image then changes into the original object.

OF SPHERICAL PROJECTIONS.

In projecting the ſphere in plano, the geometrical and ſtereographical projections are introduced, and applied ſometimes ſingly, and ſometimes both together; thus, ſuppoſe the eye to be placed in one of the poles or extreme points of the axis of the ſphere, and the plane of projection to be perpendicular to that axis, and paſſing through the oppoſite pole or other extreme thereof; in this caſe, the whole ſphere will be included between the eye and the plane of projection; and therefore the ſecond caſe only is here applied.

But the eye continuing in the ſame poſition, and the plane of projection ſuppoſed to paſs through the equator, or any of the leſſer circles parallel thereto, as the tropics, polar, or parallels of latitude, &c. then a due application of both the firſt and ſecond caſes is requiſite; for the projections of thoſe parts of the ſphere, which are between the eye and plane of projection, are derived from caſe II. and thoſe which are beyond that plane, from caſe I. Becauſe the eye is here ſuppoſed to be in one of the poles of the ſphere, it is manifeſt, from the above, that the ſaid pole can not be repreſented in the projection, by reaſon thoſe parts of the ſphere, which are neareſt the eye, will be projected fartheſt out, and recede infinitely from the center of the projection, proportionably to their vicinity to the eye; whence theſe kind of projections are generally confined to ſome particular circle of the ſphere, taken at pleaſure, according to the parts which the projection is to deſcribe, for then none of the parts between that circle and the eye are delineated on the plane of projection. Now, if the plane of projection paſs through that particular circle,

the whole projection is perspective, and comes under case I. In all these instances the projections are similar one to another, and differ only as being one greater or lesser than the other, according as the plane of projection is supposed to be more or less distant from the eye.

Having thus described the nature of mathematical projections in general, I now proceed to consider case I. more particularly, as being the principal design of this little tract, referring the spherical projections, and what I have prepared for illustrating and explaining that subject more fully, to another opportunity.

OF THE DIFFERENT APPELLATIONS GIVEN TO PERSPECTIVE.

Authors have considered and distinguished Perspective variously. Some according to the different positions which the picture may have with respect to the ground; thence denominating it to be direct, inclining and horizontal, according as the picture be supposed either perpendicular, oblique, or parallel to the ground. Others from the different positions of the picture with respect to the eye, as being either directly in, above, below, or on one side of the axis thereof. And others, again, by the different distances of the eye from the picture. But no advantage has accrued to the art from any of these varieties, there being no essential difference in the practice, whether it result from one or the other of those considerations, for the rules serve alike in all of them, let what will be the position of the eye and the picture, or that of the picture with respect to the objects.

For it is only from the different form which the picture may be of, that any essential difference can arise, as whether it be a plane, concave, or convex surface, &c. Such are the cases of appearances

defcribed on cupolas, arched cielings, uneven walls, or theatrical fcenes, &c. which laft is commonly called Scenography, and confifts in making the defcription upon feveral different planes, differently pofited, and at different diftances with refpect to the eye. And fometimes objects are reprefented by reflexion, by fo drawing them on the picture, that although they then appear confufed, and without any veftiges of a regular defign, yet, by placing a polifhed piece of metal or glafs, of a fuitable form, and in a proper pofition, the feveral parts of the picture will be thereby duly reflected to the eye, in their proper fituations, and the true image of the original, from whence they were drawn, will be perfectly exhibited to view, however confufed and unintelligible it feemed before. For, whether the rays which compofe an image be reflected, refracted, or any other way difturbed from proceeding in a rectilinear courfe to the eye, if they be fo fubjected by any contrivance, as that they at laft duly unite in the eye, they will thereby difcover the true image of that object; but thefe are fpeculative matters more for curiofity than real ufe, fhall not therefore digrefs farther on this head, but leave the reader, who is defirous to learn this particular fpecies of Perfpective, to the directions of a treatife, compofed by a jefuit, intitled, LE CURIEUSE PERSPECTIVE.

Some authors have alfo introduced a third kind of projection, called by them MILITARY PERSPECTIVE, or geometrical elevation, by which they pretend to reprefent length, breadth and depth to the view all at the fame time, by raifing the fides of the objects from their ichnography or geometrical plan, adhering to the true meafures of thofe fides, regardlefs of their different diftances from the eye, but varying the angles of elevation by the laws of ftereographical projection. The effects of this unfit mixture of both kinds of projection, is an inconfiftent medley, both unnatural and difa-

greeable to the eye, and can anſwer no purpoſe but what may be better attained by true Perſpective, is therefore unworthy of a place in this tract; but a more particular account thereof may be ſeen in the firſt volume of the jeſuit's Perſpective, printed at Paris in 1679, to which we refer thoſe who would learn this kind of projection, which we here think not worth our while to explain any farther.

II. OF THE SEVERAL PLANES NECESSARY TO BE CONSIDERED IN DETERMINING THE APPARENT POSITIONS OF OBJECTS ACCORDING TO THEIR RESPECTIVE SITUATIONS.

The fundamental principle, upon which all manner of projections depend, conſiſts in knowing how to determine or mark out, upon any given plane, the place of any point of a viſible object, as ſeen by the eye in any given poſition. Now the apparent place of any point of the object will be in that point of the given plane where a line, drawn from the eye to the ſaid point, interſects the ſaid plane, or when the given plane is beyond the object, in that point where that line duly produced meets the ſaid plane: and the poſitions of any points being obtained, thoſe of lines are eaſily derived, as being terminated by points; ſurfaces, as bounded by lines; alſo ſolids, as being contained under one or more ſurfaces; wherefore the rules of projection have their origin from hence. But the real poſition of a point in abſolute ſpace can not be known otherways than by comparing it with other objects, whoſe ſituations are given; whence ſeveral planes, differently ſituated one to the other, muſt be aſſumed, ſo that, by means of one of them, the poſition of the object may be compared with reſpect to its being even with, above, or below the eye, and by help of another, whether it be in the ſame right line with the eye, or on either ſide thereof. Thus aſtronomers ſup-

pose a plane, which they call the visible horizon, to pass through the eye, and extended every way till it coincide with the heavens; by this they distinguish the positions of the moon, stars, &c. either as being in this plane, or as being elevated or depressed above or below the same; also another plane, in like manner extended, they suppose to pass through, and at right angles to the former, and thereby distinguish objects as being to the east or west, or on the right or left hand of the observer; and many other positions relating to the sun, moon, stars, &c. are ascertained from such like imaginary planes, as is made appear in the astronomical science; so likewise, in Perspective, three planes are necessary for determining the true appearances and positions of objects.

ILLUSTRATION. Plate IV. Fig. 5.

Raise up the plane D C, and make the plane O P pass through the same, then raise up the plane S T; bring D, in the plane P D, to coincide with D in the plane D K, and make the plane L B pass through both the planes O P, S T; again, bring D, in the plane D Y, to coincide with D, and make the plane M N also pass through the said two planes O P, S T.

Now, suppose the plane S T to be an original plane, and AGBF to be an object seen in that plane by the eye at E, and let L B be a plane passing through the eye, and cutting the object, so that the line E C, drawn from the eye to the object, be perpendicular to the plane containing that object; let now the plane O P be placed somewhere between the object and the eye, and let M F be another plane, intersecting the two other planes at right angles, then will the plane L B shew what parts of the object are above, what below, and what are even with or in the plane of the eye; the plane M F shews what parts thereof are on the right, and what on the left

hand side thereof; and the plane O P shews the perspective appearance or image thereof, when projected by lines drawn through the same, from the several points of the object to the eye, as seen in that position.

Now, if L B be considered as a level or horizontal plane, cutting the picture or plane O P perpendicularly in the line a b or x y, that line is called the horizontal line; and if the plane M F be considered as a perpendicular or vertical plane, cutting the same in the line H I, this line will be called the vertical line; now the vertical and horizontal planes intersect each other in the eye's axis, or line E C; whereof that part E c, which is intercepted between the eye and the picture O P, is called the principal ray, or distance of the eye from the picture. Now, when the three planes O P, L B, M F, are given in position, the horizontal and vertical lines are also given in position on the plane of the picture; wherefore if E, the place of the eye in the intersection of the horizontal and vertical planes, be known, that is, if E c, the distance between the eye and the picture be given, and the position of any point, suppose B, be determined from any known distances thereof from the three before-mentioned planes, its position or place on the picture may from thence be easily obtained: thus, draw B C perpendicular to the vertical plane M F, and C G perpendicular to the horizontal plane L B, then will those lines measure the distances of the point B from the said two planes; then, drawing B E, C E, G E, the figure B C G E will be a triangular pyramid, having the plane of its base perpendicular to the vertical plane, and parallel to the section b c g, made by the plane of the picture O P; now, seeing the line C c expresses the distance of the said plane of the base from the picture, it is also the distance between the picture and the point B; and, because the two pyramids B C G E, b c g E, are similar,

$$E C : E c :: G C : g c,$$
$$\text{and } E C : E c :: B C : b c.$$

COROLLARY.

Hence it appears, that as the diſtance between the eye and any original object is to the diſtance of the eye from the picture, ſo is the diſtance of the ſaid object, from the horizontal plane, to the diſtance of its image on the picture from the horizontal line. Alſo, as the diſtance between the eye and the original object is to the diſtance between the eye and the picture, ſo is the diſtance of that object, from the vertical plane, to the diſtance of its image on the picture from the vertical line, &c. And if the original plane be between the eye and the picture; thus, ſuppoſe now the plane S T to be the picture, and O P the original plane, then making the difference between E C and E c, the firſt term of the proportions, the reſt will be the ſame as in the two preceeding analogies.

OF THE DIFFERENCE BETWEEN THE TWO WAYS OF CONSIDERING AND APPLYING PLANES, AS USED IN THE OLD AND NEW METHOD OF PERSPECTIVE.

Becauſe the poſition, in which the picture is moſt frequently conſidered by painters, when not concerned with works to be exhibited on cielings, &c. is that of being ſituated perpendicular to the ground or plane of the horizon, it has occaſioned the horizontal plane or ground itſelf to be conſidered as the principal, and oftentimes as the only plane neceſſary to be referred to; thus we ſee the horizontal plane called the geometrical plane, ground, floor or pavement, &c. and the line, in which the picture interſects the ſaid plane, when thus conſidered, called the ground line; now, when the ground and picture are perpendicular to each other, what we call

the vaniſhing line of the ground will then paſs through the center of the picture, (theorem IX. part II.) and the center of the picture will, in this caſe, be the center of that vaniſhing line, (def. XIX.) This vaniſhing line the writers on the old perſpective call the horizontal line, as being the apparent boundary of the viſible horizon conſidered as a level plane, as was before obſerved in the ſcholium to theorem IX. part II. Now, ſeeing the poſitions of objects, whoſe perſpective repreſentations are required, are many of them placed on the ground, and elevated perpendicular thereto, as the front and ſides of buildings, &c. ſuch objects may therefore be conceived, as being in planes perpendicular to the ground; whence the vaniſhing lines of thoſe planes will, in ſuch caſes, be perpendicular to the line thus denominated the horizontal line, and the center of thoſe vaniſhing lines will alſo be in the ſaid horizontal line. (cor. I. theorem XVII. part II.) When the picture, as thus conſidered, is ſo ſituated with reſpect to the original objects, as that they have one line or ſide thereof, as the front of a building, &c. parallel thereto; and the other lines or ſides thereof, as the ſide fronts, &c. perpendicular to the picture; then the vaniſhing lines of thoſe ſide fronts, as well as the horizontal line, will paſs through the center of the picture, which center will be the common center of both thoſe vaniſhing lines, (cor. II. theorem XVII. part II.) and the vaniſhing point of the common interſection which ſuch planes make with the ground, as well as that of all other lines, in any of thoſe planes, parallel to their interſection with the ground, will alſo be in the center of the picture (cor. I. of the ſame); whence it appears, that the center of the picture is of conſiderable uſe in determining repreſentations on the picture, when thus ſituated with reſpect to the original plane, and is therefore called the principal point, or point of ſight; but, in the method we eſpouſe and wiſh to recommend,

the point of ſight is that which is deſcribed in def. IV. part II. and the axis of the eye being the line which produces the vaniſhing point of all planes perpendicular to the picture, it is therefore the line which produces the vaniſhing point of all thoſe planes which moſt frequently occur in this ſituation of the picture; whence it is called the principal ray, and the length of this line being ſet off from the principal point, both ways upon the horizontal line, will give two points, called the points of diſtance. Now, in this particular ſituation of the picture, with reſpect to the original objects, the principal rays being either perpendicular or parallel to the ground or to the picture; they will therefore make the ſame angle with the ground line, as the original lines themſelves make with the ground, (cor. IV. theorem IV. part II.) and thoſe lines which are parallel to the ground, but not to the picture, will have their vaniſhing points in the horizontal line. (cor. IV. theorem IX. part II.) Now, when theſe points fall beſide the principal point, they call them accidental points in the horizontal line; and when it is required to find the repreſentations of lines, which are neither parallel nor perpendicular to the picture, nor to the ground, they generally ſeek their images, by having recourſe to the ſeats which ſuch lines have on the ground, for thereto they moſt commonly refer all ſuch lines as do not come within the reach of the above deſcription, without enquiring after their vaniſhing points, or the lines which ſhould produce them. But theſe principles are too confined for general uſe, and can ſerve only in this particular circumſtance, of conſidering the picture as being perpendicular to the ground; whereas the practice of this art requires that any other plane, and not the ground only, may be taken as an original plane, and the picture may have other ſituations with reſpect to the ground, or the original objects, ſuch as being parallel thereto, inclining to,

or reclining from the ſame, &c. for the parts of a building, as roofs, &c. are not always either parallel or perpendicular thereto; and there are alſo caſes which render it neceſſary to conſider, in the ſame picture, ſeveral different original planes, each having its own interſecting, vaniſhing, and the ſeveral other lines and points peculiar thereto, by means whereof many operations are eaſily performed, which would otherwiſe be extremely tedious and difficult, if not, in ſome caſes, impracticable; but, that we may candidly favour this old ſyſtem of Perſpective, as far as propriety will permit, will allow the horizontal line and ground line to be proper terms for expreſſing the vaniſhing and interſecting lines of the ground, when that is taken for the original plane, but think it abſurd to give theſe appellations to any other plane than the ground; for the original plane may be ſuch, that the vaniſhing line thereof ſhall interſect the picture either in, above or below the horizontal line, or it may be that it croſs or interſect the ſame, as may be thus exemplified.

PLATE VI. Fig. 7.

Bring A to coincide with A, make the planes G P, S L, paſs through each other in the line T R; make K, in the plane K L, coincide with K in the plane K A or K B, paſs the plane F M through the plane K L, in the line V M, and paſs the plane M H through the plane S L, in the line H L. Now, let the plane G P repreſent an original plane, inclining to the picture S L; let the plane F M be conſidered as an horizontal plane, paſſing through the eye at E, and interſecting the picture in the line F N, then is the line F N the horizontal line: let now a plane, as M H, paſs through the eye, and be parallel to the ſaid original plane G P, cutting the picture in the line H L, then is the plane M H the vaniſhing plane of the original G P, (def. VIII.

part II.) and H L the vaniſhing line of the ſame, (def. XI. part II.) conſequently the vaniſhing line falls above the horizontal line in this caſe; and if the original plane had declined from the picture the contrary way, the vaniſhing line thereof would then have cut the picture in a line below the horizontal line, as is eaſy to conceive from what has been here ſhewn, and when the original plane meets the picture at right angles, the horizontal and vaniſhing lines thereof coincide, or fall together in one and the ſame line only; ſuppoſing, as is here done, that one ſide of the original be parallel to the picture, for, if all the ſides thereof are oblique thereto, its vaniſhing and horizontal lines will then interſect each other, as was ſhewn in theorem XVII. part II. the vaniſhing line H L in that figure being now the horizontal line.

The ſeveral efforts, excluſive of the laſt, which have been made in oppoſition to the endeavours uſed for conveying and eſtabliſhing a general knowledge and uſe of this new method of Perſpective will, I hope, ſufficiently juſtify me in this attempt, to ſhew the truth and univerſality thereof; and thus endeavouring to ſet learners right in their firſt notions of the different methods here alluded to, by warning them of the defects under which this art will labour, when diſrobed of the improvements added to it, by conſidering the poſitions of planes in a general view, and thence making the neceſſary practical applications ſuitable for obtaining the advantages it offers to us.

OF THE POSITION OF THE PICTURE WITH RESPECT TO THE ORIGINAL OBJECTS.

Although the picture may have any poſition with reſpect to the objects which are to be repreſented thereon; yet, in order that it may ſhew an agreeable and natural repreſentation of the things deſcribed, it ſhould be ſo placed, that the ſeveral objects may appear

in their natural ſituation, that is, ſuch objects as would really be ſeen by rays parallel to the horizon, or by rays inclining upwards above the ſame, or by others declining downwards below it, may appear on the picture by rays in ſimilar correſpondent poſitions; for the ground or plane of the horizon being the natural or apparent ſeat of all viſible objects, whether terreſtrial or celeſtial, the mind judging them to be higher or lower, according as they are more or leſs elevated above it; the ſituation of all objects may and ought therefore to be referred to that plane, and have ſuch poſitions given them in reſpect thereto, as is agreeable and conſiſtent with their natural appearance; wherefore the moſt proper diſtinctions, to be made of the different ſituations which may be given to the picture, ariſe from its poſition with reſpect to the ground or plane of the horizon, and are therefore three, perpendicular, parallel, or inclining.

OF THE PERPENDICULAR POSITION.

The perpendicular ſituation of the picture is beſt adapted for repreſenting the ground itſelf, and the ſeveral objects which are erected thereupon; and this being a poſition which is parallel to the uſual poſture of the body of the ſpectator wherein the eye is moſt uſed to view original objects, it follows, that the repreſentations hereby formed on the picture, appear the moſt natural and agreeable to the originals themſelves. In this poſition the ichnography of the deſign on the ground is conſidered as deſcribed on a plane perpendicular to the picture, the vaniſhing line whereof paſſes through the center of the picture, (theorem X. part II.) and is parallel to the horizon, and alſo repreſents the ſame, is therefore, in this caſe, called the horizontal line; the elevations of upright objects are conſidered as being in planes perpendicular to the ground, which ſaid planes may be either perpendicular, parallel or oblique to the picture, but their

vaniſhing lines are always perpendicular to the horizontal line, (cor. II. theorem XVII. part II.) and all lines, which meaſure the perpendicular altitudes of objects, thus ſituated above the ground, are parallel to the picture; but it muſt be here obſerved, that the ground, deſcribed in the picture, is ſuppoſed to be level or truly horizontal; for, if the ground have aſcent or deſcent, the picture continuing perpendicular to the horizon, the vaniſhing line of the ground will not, in either of thoſe circumſtances, coincide with the horizontal line, but will interſect the picture either above or below the ſame, according as the ground is elevated or depreſſed; but, nevertheleſs, the lines, which meaſure the perpendicular heights of the original objects, will yet be parallel to the picture, as being perpendicular to the horizon, and they, with reſpect to the ground, will repreſent the oblique ſupports of the ſeveral points above that inclining plane.

The poſition of the picture here deſcribed is proper to landſcapes, views, buildings, hiſtorical pieces, and in general to all pictures where the ſpectator is ſuppoſed to ſtand on the ground, and having the axis of the eye directed parallel thereto.

OF THE PARALLEL POSITION.

Here are two caſes. 1. When the eye is between the ground and the picture. 2. When the picture is between the eye and the ground.

In the firſt, the axis of the eye is ſuppoſed to be turned perpendicularly upwards, and conſequently the ground, being behind the eye with reſpect to the picture, can not have any part thereof repreſented on the picture; whence all kinds of terreſtrial objects are here excluded, excepting ſuch as may be ſuppoſed to aſcend above the plane of the picture, ſuch as the higher parts of mountains or

buildings, or ſuch other objects as may be conceived in the air, but all their uppermoſt ſurfaces muſt neceſſarily be concealed from the eye.

Pictures or paintings on flat cielings are of this kind, and thoſe on cupolas or arched roofs may be reduced hereto, they all agreeing with reſpect to the particular kind of objects proper to be deſcribed thereon. A neglect in giving due attention to this particular circumſtance, has rendered ſeveral performances of this ſort diſagreeable and unnatural; but a judicious painter will forbear to repreſent on a picture, in this ſituation, the ſea, ground, part of the floor of a building, or the upper faces of ſteps, &c. notwithſtanding examples of ſuch inconſiſtencies are to be met with.

In the ſecond caſe, the eye is ſuppoſed to be at ſome height above the picture, and its axis turned perpendicularly downwards; in this circumſtance no part of the ſky can appear, nor any other thing but what can be ſuppoſed to lie either upon the ground, or between it and the picture, ſuch as the pavement or floor of a building, the plan of a garden, &c. and thoſe parts of the building, or ſuch other things which, ſtanding upon the ground, do not reach up to the picture; whence this ſituation of the picture is the moſt confined of any, and ſeldom uſed but out of curioſity, as on the floor or pavement of a church or dome, which, by an artful diſpoſition of materials differently coloured, may be made to repreſent objects proper to that ſituation, or even the reflected image of the building itſelf, as appearing in a looking glaſs, or ſtagnant water, to an eye viewing it from ſome convenient place, as a gallery, &c. at the top or upper part of the building.

In both the caſes here mentioned, the ichnography of the objects on the ground is deſcribed as on a plane parallel to the picture, and is therefore ſimilar to its original; the planes of the elevations, and

the lines, which meaſure the perpendicular altitudes of the objects above the ground, are perpendicular to the picture; wherefore the vaniſhing lines of thoſe planes paſs through the center of the picture, which center is the vaniſhing point of all the ſaid lines of altitude; now, ſeeing the ground can not interſect the picture in either of thoſe two caſes, it is not proper to be uſed as the principal original plane, there being no horizontal line in thoſe caſes, but any of the upright ſides of the building deſigned to be repreſented, or any other ſubſtituted plane perpendicular to the picture, may be uſed for that purpoſe, and the objects required to be repreſented may be thereto referred.

OF THE INCLINING POSITION.

The various inclinations, under which the picture may be placed with reſpect to the plane of the horizon, are innumerable, which will have correſponding effects on the place of the horizontal line and the vaniſhing point of the perpendiculars to that plane; as to the kind of objects proper to be repreſented on a picture of this ſort, they are to be determined from what has been delivered concerning the two poſitions before mentioned, and will differ according as the inclination of the propoſed picture approaches nearer to one or to the other of them; but theſe oblique poſitions are ſeldom practiſed, except neceſſity obliges it, from the particular poſition of the wall or place in which the picture is to be painted or placed.

SCHOLIUM.

Let the poſition of the picture be what it will, if it be ſuch that the horizontal line can appear therein, it ought, in ſtrictneſs, to be ſo placed as that the eye, when in the true point of ſight, may be on a level with that line; for then all the repreſentations, formed

on the picture, will appear as being in their true and natural positions. This may, however, be dispensed with, in some measure; for if a picture be drawn, by supposing it perpendicular to the horizon, and it should be necessary to place it so high, that the axis of the eye can not reach up to a level with the horizontal line, then the true appearance which the picture ought to exhibit may be preserved, by inclining the picture forwards, so that a line may be drawn or conceived to issue from the eye perpendicularly thereto, and meeting the same in the center; for, although the ground represented in the picture does not then appear parallel to the horizon, but as rising upwards, yet the spectator, in viewing and considering the representations according to the relations which they have one to the other, without referring to their true positions, or his own real posture, with respect to other objects which are out of the picture, will readily pass over the defect of this deviation from the true position, so long as the whole picture is every where consistent with itself; and the picture being in this situation will, in some sort, exhibit a resemblance of the appearances which the like objects would shew, if reflected by a well-polished plane mirror, or common looking-glass, placed inclining to the horizon, which makes the reflected ground appear as being elevated or depressed with respect to the real ground; but the reflections of all other objects preserve the same relations with respect to the reflected ground, which the real objects have to the ground itself: for, let the spectator but only imagine that he stands perpendicular to this reflected ground plane, and then all will appear as having a natural situation with respect to the ground on which they are supposed to be placed.

But let it be observed, that this liberty be not used too freely; it is allowable only to detached pieces, which are to be placed where conveniency suits; for, such as are expresly designed for a certain

fixed situation, as a wall, cieling, &c. do not admit of this variation from the true position, by reason such pieces have generally a more immediate relation to the rest of the building itself. Suppose, for example, that a picture, designed for a position which is perpendicular to the horizon, were placed in one that is parallel thereto, as a cieling, &c. the spectator will not then be able to conceive it as exhibiting representations of objects according to their natural appearance, without imagining himself to be standing erect on a plane which appears perpendicular to the horizon, a conception which requires such a force of imagination as is not easily acquired, nevertheless, we have some examples which render this extraordinary effort of the mind absolutely necessary, before the picture can possibly have its due effect.

OF THE DISTANCE OF THE EYE FROM THE PICTURE.

From what has been advanced on the nature of stereographical projections, it is manifest that the representations, formed on the picture, can not appear to the spectator exactly as they ought to do, if the eye be not in the true point of sight; whence it is evident that they should, in strictness, be always viewed from that point; therefore a true determination of the position of the eye is such an essential requisite in this art, that no examples or authority whatever should induce us to disregard it, or be too careless about the choice of it.

In a room or place that is bounded on all sides, whatever be the position of the picture, such a distance must be taken for the eye as is within reach; thus, suppose the picture were to be on the cieling, the distance is then determined by the height of the room, deducting therefrom the height of the spectator's eye from the floor; but,

when the place is not thus circumscribed, the artist is more at liberty to make such a choice for this distance as shall be most suitable to the grand design of making the objects appear to the best advantage, and in their natural situations. Now, when the picture is in a perpendicular position, the artist has generally the greatest room for exercising himself in making his choice of a proper distance to work from; let us therefore briefly enquire into the effects which are produced from different distances of a picture in this position, for, what we shall learn from hence, may be easily applied to any other position of the picture whatever. In this enquiry I suppose the height of the eye to be given or taken at pleasure, and remaining constantly the same. The eye thus abstractedly considered, being placed at different distances from the picture, will produce correspondent changes in the ichnography and orthography of the original objects; for the ichnography thereof will be affected either with respect to the quantity of the whole space which it contains, or the proportions of that space taken up by its different parts, according as they are nearer to or farther from the picture, and also as to the apparent breadths of those parts. Now here are two things which ought to be carefully guarded against: first, that the apparent decrease of equal parts in the ichnography may not be too sudden, as they become more remote. Secondly, that such remote parts as are to be distinctly expressed, may not approach too near the vanishing or horizontal line; for if they do, their images will be too small, and too much crouded together, both which contribute to render them indistinct, and the picture will be thereby defective; which imperfections are avoided by making a right choice for the place of the eye, or the distance thereof from the picture.

L E M M A I.

The distance between the image of any point given in an original plane, and the vanishing line of that plane, is to the vertical line thereof, or to the director, as the line which produces the vanishing point of a line drawn in the original plane through the said given point, is to the distance between the said point and the directing line.

ILLUSTRATION. Plate XI. Fig. 13.

Bring D, in the plane D B, to coincide with D in the plane N B; make S, in the plane S H, coincide with S in the plane N B; make the plane Q B pass through the plane S H in the line F G, and pass the plane Q Z through the plane S H, in the line line H L.

Let A be the point given in the original plane N B; through the same draw the line A B parallel to the intersecting line S I; let Q Z be the vanishing plane of the original plane N B; from the directing point D to any point, as K, in the line A B, draw the line D K, and parallel thereto, from the point of sight, draw the line E C; then is C the vanishing point of the line D K, and E C its distance, P C the vertical line, E D the director, M N the directing line, &c. draw also the lines E A, E B, E K: now A B, being parallel to S I, its image a b is also parallel to S I; and K being a point in the line A B, and k the image thereof, the line a b is that which must contain the image of A, as has been before shewn; but the distance of a, from the vanishing line H L, being equal to k C, and the distance of A from M N is equal to D K, because E C and P C remain constantly the same, it follows, that in what point soever of A B the point A be taken, the distance between the image of that point and the vanishing line will be to P C as E C is to the distance between the given point and the directing line.

COROLLARY I.

The distance between the image of any point in an original plane and the vanishing line remains the same wherever the point of sight be taken in the parallel of the eye; for A B, a b and X Q being all in the same plane, a line drawn from any point in X Q to any point in A B, must cut the picture somewhere in the line F G, and all points in that line are equally distant from the vanishing line H L.

COROLLARY II.

If the height of the eye be increased or diminished, the eye being in the same directing plane, the distance between the image of an original point and the vanishing line will be increased or diminished in the same proportion.

COROLLARY III.

If the height of the eye remain constantly the same, and the distance thereof be varied, the distance of the image from the vanishing line will vary accordingly.

LEMMA II.

The difference between the image of a nearer part of an original line and the image of the part next beyond it, is greater than the difference between this last image and that of the next succeeding part, and so on of others.

PLATE VIII. Fig. 9.

For, let O be the point of sight, the parts A K, K G, G Y, being equal, q n : n m :: D G : D A, by cor. III. theorem XIX.

and for the same reason n m : m y :: D Y : D K,

also q n — n m : n m :: D G — D A = A G : D A,

$nm - my : my :: DY - DK = KY$ or $AG : DK$.

But AG is greater in proportion to DA than it is to DK, therefore $qn - nm$ is alſo greater in proportion to nm than $nm - my$ is to my; and nm being greater than my, therefore is $qn - nm$ ſo much the more greater than $nm - my$.

COROLLARY I.

The more remote that the equal parts of the original line are from the directing point, the nearer do the images of thoſe parts approach to an equality, by reaſon their differences are accordingly diminiſhed.

COROLLARY II.

The greater the diſtance of the eye, the more nearly equal do the images become of any two adjacent parts of the original line.

For, ſuppoſe the eye removed from O to E, then, becauſe A K and K G are equal, the image of A K is to that of K G from the point of ſight O, as N G to N A, and, from the point of ſight E, they are as D G to D A; but D G is leſs in compariſon to D A, than N G is to N A; therefore the image of A K is leſs in proportion to that of K G, from the point E, than it is from the point O: but a k, k g, the images of A K and K G, are in the ſame ratio one to the other wherever the eye be placed in the ſame directing plane; therefore, whether the eye be placed at E, or any where elſe, in a plane paſſing through E parallel to the picture, the images of A K, K G, ſeen from thence, will be nearer equal to each other than they would be if formed by viewing them from O.

COROLLARY III.

If, from the interſecting point F of any line F Y, ſeveral diſtances, F A, A K, K G, G Y, &c. be taken, each equal to N F, the

point N being the directing point, q C, n C, m C, y C, the images of those parts, will be in proportion to O N, or F C, as the following fractions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c. and the images of the parts themselves will be as $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, $\frac{1}{30}$, &c. of F C, the differences of the denominators of each of these last fractions increasing from o in this series 2, 4, 6, 8, 10, &c.

In the similar triangles ONA, q F A,

it is $NA : FA :: ON = FC : Fq$,

but $NA = 2\,FA$ by hypothesis; whence $FC = 2\,Fq$,

consequently $Fq = qC = \frac{1}{2}\,FC$.

Again, in the similar triangles O N K, n F K, we have

$NK : FK :: ON = FC : nF$,

but $FK = \frac{2}{3}\,NK$; whence $nF = \frac{2}{3}\,FC$,

consequently $nC = \frac{1}{3}\,FC$,

and in the same manner it may be proved, that m C is $\frac{1}{4}$ F C, and $yC = \frac{1}{5}\,FC$.

Now, if we call $FC = 1$, $qC = \frac{1}{2}$, $nC = \frac{1}{3}$, $mC = \frac{1}{4}$, $yC = \frac{1}{5}$, the image n q, which is the difference between q C and n C, will be $\frac{1}{6}$, $nm = \frac{1}{12}$, $my = \frac{1}{20}$, &c. the difference of the denominators of which fractions increase from o in the series 2, 4, 6, 8, &c.

Suppose now the parallelogram S Y to represent part of the ichnography of the original design, subdivided into smaller ones as in the figure, N F being chosen for the spectator's line of station, which determines the position of the ichnography with respect to the picture; take any height at pleasure, as N O, for the height of the eye, supposed now in O, and C O its distance from the picture; then, if the images q, n, m, y, of the points or divisions A, K, G, Y, and consequently the distances of the images of the cross divisions of the ichnography, passing through those points, shall appear too unequal, or fall off too suddenly; or that y, the image of Y, the most remote

division, appears too far up in the picture, or too near the vanishing point G, the fault may be removed by inlarging the distance of the eye, and fixing it in some other point of the line C O produced, suppose at E, &c. Now, when C O, the distance of the eye, is taken equal to F A, the distance of the first division from the picture, the heights of the images q, n, m, y, above the intersecting line, will be $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c. of F C, the depth of the original plane, their respective distances from the vanishing line being $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c. of that depth, and the images of the parts F A, A K, K G, G Y, &c. will be $\frac{1}{2}$, $\frac{1}{6}$, $\frac{1}{12}$, $\frac{1}{20}$, &c. of that same depth; but if, at this distance of the eye, the parts near the picture occupy thereon a greater space than is agreeable with respect to what is possessed by the other more distant parts, or that the objects, situated between G and Y, appear too small and crouded, the distance of the eye may be taken equal to the two first parts F A and A K together, which will bring the images of the divisions, next succeeding those here mentioned, down to the same points on the picture which they themselves were in before; and thus may there be found such a distance for the eye, as shall make any point in the ichnography to appear at any proposed height on the picture, within the limits of the depth of the original plane; but, if it were required to represent any particular part of the ichnography, suppose that between G and Y, such as shall occupy the greatest space in depth possible, the distance of the eye must then be a mean proportional between the two parts F A and F Y; for, at that distance, the space A Y will occupy a larger field in depth than it would do at any other distance of the eye in the line C O, produced either nearer to or farther from the picture. Hence it appears, that the distance of the eye is that which principally commands the distance of the most remote ground that can be described with any tolerable distinctness; for, if the ground be-

yond the picture be divided into ſpaces equal to the diſtance of the eye, the ſeats of all ſuch objects as are contained in the ninth ſpace from the picture, can then occupy no more than the one-ninetieth part of the depth of the original plane, and the image of the extremity of that ſpace being but one-tenth part of that depth diſtant from the vaniſhing line, that one-tenth part is therefore the whole ſpace that is left wherein all ſpaces beyond the ninth can poſſibly be repreſented on the picture, as appears from what has been premiſed, for there being now nine diviſions, the firſt fractions are

$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$;

the ſecond $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, $\frac{1}{9}$, $\frac{1}{10}$;

and $9 \times 10 = 90$.

E X A M P L E.

Suppoſe the height of the eye to be 5 feet, and its diſtance from the picture 20 feet.

Then the ſeats of all ſuch objects as are between 160 and 180 feet diſtance from the picture, can poſſeſs no more ſpace than one-ninetieth part of 5 feet, or two-thirds of an inch, and this reaching within 6 inches of the vaniſhing line, or, which is the ſame, the tenth part of 5 feet, it follows, that the ſeats of all objects on the ground, from 180 feet to any aſſignable diſtance, muſt be confined within that 6 inches, ſo that, even at ſo great a diſtance of the eye as 20 feet, the ſeats of objects, 20 feet in depth at the diſtance of 60 or 70 yards, can be repreſented but imperfectly, and all beyond that diſtance will be faint and confuſed; and, if the diſtance of the eye be leſſened, the ſpace which can be diſtinctly deſcribed will be proportionably decreaſed. There is alſo another effect which the diſtance of the eye produces upon the ichnography; for, as by enlarging the diſtance the images of the ſeveral croſs diviſions are

brought lower down towards the intersecting line, so their apparent measures are proportionably increased, and consequently the lines, which measure the breadths of objects parallel to the picture, appear longer, at the same time that those, which measure their depths, become shorter; and, as the apparent breadths of objects are thus encreased, so also are their apparent heights or elevations, those heights being supposed parallel to the picture; so that, upon the whole, supposing the picture and original objects as retaining their position, if the distance of the eye be enlarged, their apparent breadths and heights, or those sides thereof which are parallel to the picture, will be thereby encreased, but their depths will be diminished. Now, although we may, from what has been said, easily find such a distance for the eye as is suitable for representing the objects in the most advantageous situation on the picture, so as that the remotest parts may not appear advanced too near the horizon, nor their depths decrease too suddenly, but that a due and agreeable proportion between their apparent heights, breadths and depths may be preserved in the representations formed on the picture, yet, let it be observed, that what has been advanced on this subject, is to be understood as relating to a picture, formed from a scale, equal to the true measures of the original objects, that is, when the objects adjoining the picture are represented big as the life; but, as this scale may be contracted to any dimensions at pleasure, it follows, that a picture may be rendered capable of representing much greater distances than the above-mentioned, and yet the distance of the true point of sight may be greatly lessened, thus,

PLATE XI. Fig. 13.

Make D, in the plane D K Q, coincide with D in the plane N B, and pass the plane S H through the same; make B, in the plane

B R, coincide with B in the plane N B; then make the plane Q Z pass through both the planes S H and B R.

Now let A B M N represent the ground plane on which the ichnography of the intended objects is supposed to be formed in its true dimensions, and let A B be the intersection of the ground with the supposed picture A B Z R, and consequently the nearest part of the ground proposed to be represented on the picture; now, if D K or E O be the distance of the eye necessary for rendering the representations sufficiently distinct, and this be too great for the place where the picture is to be fixed, it must be drawn from a lesser scale, which may be thus determined.

Between the directing point D and the intersecting line A B draw in the original plane a line, as S I, parallel to A B, on that line erect a plane parallel to the plane or picture A B Z R, and upon the plane thus erected find the perspective representation of the given plane, the eye being at E, which is here expressed by a b z r; then, if a b z r be taken for the picture, E C for the distance, and E d for the height of the eye, and that the true measures of the objects to be represented be all reduced to the same proportion as a b bears to A B, then all objects, described in the picture a b z r, with those proportional measures with the last-mentioned distance and height of the eye, will be exactly similar to a picture of the same objects, drawn on the plane A B Z R, according to their true or real dimensions, having E O and E D for the distance and height of the eye, but as much lesser as is the proportion assigned between a b and A B. Thus may the scale, and consequently the distance of the eye, be reduced to any convenient measure to suit the situation in which the picture is to be placed.

The space between a b and A B is entirely hid in this case, no part thereof being to have a place on the picture; and, as A B is the true

intersecting line of the ground on which the true measures of the objects should be taken, if that were the place of the picture; so a b, being the representation of A B, is therefore to be considered as the intersecting line of the picture a b z r, on which the proportional measures must be used according to the diminished scale. If the picture a b z r were continued down to S I, its intersecting line with the original plane, the representation of the space between S I and a b will fall between S I and a b, and the objects situated in that space may be described either by setting off their true measures on S I as the intersecting line, or by their proportional measures on a b according to the diminished scale; for, whether the one or the other of these measures be used on those respective lines, such parts of the representation as come within a b z r will, in either case, be the same; and thus may a picture be formed that shall represent a very large and extensive prospect, the eye being at a moderate distance only, by considering the picture a b z r as a window in a second or third floor, and the spectator as viewing the objects through the same; for, in proportion to the height of a b above the ground, and the distance E C taken for the eye, the visible intersecting line A B of the ground, where the description begins, may be removed to any required distance, which will have a corresponding effect on the space which is possible to be represented distinctly. As the picture a b z r is a kind of miniature representation of the picture A B Z R, the nearest objects which are represented thereon must be less than the life, they being supposed equal to the life at A B: but, if objects were represented according to their real dimensions on the plane a b z r, and thence projected upon the plane A B Z R, they would there appear bigger than the life, which is proper to be done when the distance of the eye is necessarily very great, as in the case of pictures formed on the cieling of a church, &c. where the height is

very great, and the picture designed to exhibit the representations as being nearer than they really are, which it will do, if to this circumstance be added proper and sufficient strength of colour, for that also ought now to be encreased beyond that of real nature, in order to produce the effect here mentioned. In short, if the representations be formed from a scale less, equal to, or greater than the real dimensions of the original objects, they will accordingly appear to the spectator as being farther from, at an equal distance, or nearer than they really are when duly coloured according to those several different circumstances: but, notwithstanding this, it is not necessary, in miniature paintings, to describe the figures in that faintness of colour with which the originals would really appear when so far distant as reduces them apparently to such a size, by means of the angle under which the visual rays do then exhibit them, for a greater distinction of parts and vivacity of colour may be here allowed, taking care only that a due diminution be observed amongst the several objects represented on the picture with respect one to the other; for a picture thus drawn may still be looked upon as such a representation of the objects as would be produced by viewing them through a concave glass, which lessens their apparent magnitudes, but does not take away the distinction of their parts nor their strength of colour in so great a proportion.

OF THE HEIGHT OF THE EYE.

It appears, from what has been shewn, that the height of the eye determines the depth of the original plane, and is always equal thereto, consequently is that which gives bounds to the space which must contain the ichnography of all objects on the original plane that can be represented on the picture; that the image of a line, in a plane parallel to the picture, is of the same length wherever the

eye be placed in the directing plane; therefore the elevating or depressing the point of sight will produce no difference in the apparent heights and breadths of objects, or such of their dimensions as are parallel to the picture, for they remain of the same length, let the height of the eye be what it will, so long as its distance from the picture remains the same; also, that the images of any determinate parts of an original line, which inclines to the picture, will have the same ratio to each other at all different stations of the eye taken in the directing plane, and therefore the altering its height, without changing its distance, can have no influence on the apparent decrease of the equal parts of the lines which measure the depths or distances of the objects, by reason they have still the same proportion one to another, let the height of the eye be what it will, and are effected only as to their being greater or less in proportion to the height which is given to the eye.

DEFINITION.

A line or measure, taken upon any line parallel to the intersecting line, having the same ratio to a given original line as any determinate part of that parallel bears to its original, is called the PROPORTIONAL MEASURE of the original line on that parallel.

LEMMA.

If any determinate part of the indefinite image of an original line inclining to the picture be taken, and a proportional measure thereof be found upon a line, drawn parallel to the intersecting line, through that extreme thereof which is nearest the intersecting line, it will be as the complement of the proposed image is either equal, greater or less than the line producing its vanishing point, so will the assumed

part of that image be accordingly equal, greater, or leſs than its proportional meaſure.

PLATE X. Fig. 11.

Let D K be ſuppoſed the indefinite image of an original line inclining to the picture, and ſuppoſe the line producing its vaniſhing point to be equal to H K; let g m be the determinate part, and g f its proportional meaſure taken on a line parallel to the interſecting line, and paſſing through g, its neareſt extreme to that line; then if m K, the complement of the aſſumed part, be equal to, greater, or leſs than H K, g m will be accordingly equal, greater, or leſs than its proportional meaſure g f.

For ſeeing the triangles H K m, m g f, are ſimilar, it will be

m K : K H :: g m : g f;

therefore, if m K be equal to K H, or whether it be greater or leſſer, the ſame will g m be with reſpect to g f.

PLATE VIII. Fig. 9.

Suppoſe now O N the height of the eye, to be taken ſo great in reſpect of its diſtance O C, that the image q n of any part of an original and inclining line ſhall fall at a greater diſtance from its vaniſhing point, ſuppoſe C, than the length of the line O C which produces its vaniſhing point, then will the image n q be greater than the proportional meaſure of its correſponding original line, when meaſured on a line parallel to the picture paſſing through its neareſt extremity; but, as it muſt ſeem unnatural that the image of a line, inclining to the picture, ſhould appear equal to or greater than the image of a line, of the ſame length, parallel to the picture, and oppoſed directly to the eye, at a diſtance no greater than the neareſt extremity of the ſaid inclining line, ſuch an appearance muſt needs

be deformed and disagreeable, it ought therefore to be avoided, which it may, by taking such a height for the eye, that the indefinite image of any line in the original plane shall not be longer than the line which produces the vanishing point of that line, for then, whatever part of the original comes to be described within those bounds, its image will always be less than its proportional measure; whence it plainly appears, that the height of the eye has an immediate dependance on its distance, which it ought neither to equal nor exceed, but may be less at pleasure, according to the nature of the design, and desire of the artist for obtaining more or less room for the depths of his objects, so as shall procure an agreeable proportion between their ichnography and elevations. Nor is the place of the eye always to be considered as being the same with that of a spectator standing upon the original plane, for the eye may be conceived as placed on an eminence, at a considerable height above the original plane, such as an hill, the upper part of a building, &c. as when objects, situated below, in low grounds, or valleys, &c. are to be described.

PLATE XI. Fig. 13.

Bring D, in the plane D A B, to coincide with D in the plane N B; make B, in the plane A Z, coincide with B in the plane N B; make the plane S H pass through the plane D A B in the line F G, and make the plane Q Z pass through both the planes S H and A Z.

Let now A B M N be the original plane containing the objects to be represented, A B the nearest limits of the ground that is to be described, then may the spectator be placed in d, or any other point of the line E D, provided that the picture can be so placed as that the eye, when thus proportionally raised, may be on a level with the horizontal line Z R.

OF THE SIZE OF THE PICTURE.

What has been premiſed with reſpect to limiting the greateſt height of the eye relates equally to the ſize of the picture; for, in both, the principal inconvenience, neceſſary to be guarded againſt, is the exceſs of the image of any part of an original line above its proportional meaſure.

PLATE X. Fig. 12.

Let C be the center of the picture, E C the diſtance of the eye; from C, with the radius E C, deſcribe a circle, then if that circle, or any rectilinear figure, D F L P inſcribed therein, be made the bounds of the picture, the image of no line whatever, which is perpendicular to the picture, can, within thoſe limits, be extended farther than the radius, which is the diſtance of the eye from the picture, and therefore, when the principal lines of depth contained in the deſign are perpendicular to the picture, its ſize may be thus determined.

But, when the principal lines of depth incline to the picture, as buildings, having an oblique poſition, &c. then the vaniſhing points of the ichnography of their inclining ſides muſt be taken as centers, and the reſpective diſtances of thoſe vaniſhing points as radii, and circles deſcribed therewith, which, by their mutual interſections, will mark out a ſpace, beyond which no part of the images of thoſe inclining ſides ought to extend.

Thus, ſuppoſe V, v, were the vaniſhing points of the ichnography of the ſides of a building, then taking thoſe points for centers with the diſtance V E or v E, draw the arcs E S e, E T e, interſecting each other in E and e, and the ſpace included thereby will be that in which the repreſentation of the propoſed building ought to be confined,

These rules are to be considered as more immediately relating to the representation of pieces of architecture and animal figures, or such other objects as are of a certain known and determinate form, so as not to suffer their representations to be projected too far distant from the center of the picture; for, as to such other objects as have uncertain, variable and indeterminate forms, as clouds, hills, mountains, and such like, a greater latitude is allowable: for, although what has been before asserted, concerning the necessity of viewing a picture from the exact point of sight, be true, wherever that point be taken, yet, if the distance of that point be too small for the size of the picture, the images of objects near the sides thereof will be extended to great lengths, and take up more space on the picture, than the objects themselves would do if seen directly, and when a picture thus drawn is seen from a different point, the images of those objects will appear deformed or distorted, and be disagreeable to the eye; we should therefore adapt the size of the picture to the distance of the eye, so that none of the representations may appear monstrous or unnatural, wherever the eye be placed to view it; for, notwithstanding that it be from the true point of sight only that a picture can appear exactly as it ought to do, yet, when the distance of the eye is comparatively large with respect to the size of the picture, so that the greatest dimension of the picture may be seen under an angle of ninety degrees or less, any little deviation of the eye from its true place will not have so sensible an effect on the appearance of the picture, as when the distance is smaller, or the picture of a greater extent; and seeing pictures are frequently, if not always, placed in such positions, as that they may be viewed from different points; they ought therefore to be so drawn that, in any of the points fronting them, they may appear as little disagreeable to the eye as is possible, and if nothing contained in them appears remarkably de-

formed, little variations from the ſtrict appearances which they ought to exhibit, will be readily excuſed, and the defect ſupplied by the imagination of the obſerver.

OF THE CONSEQUENCES WHICH ATTEND THE VIEWING OF A PICTURE FROM ANY OTHER THAN THE TRUE POINT OF SIGHT.

It has been before obſerved, that the repreſentations formed on the picture can not appear ſtrictly as they ought to do, if the ſpectator be not in the true point of ſight; and the great perfection of this art conſiſts in forming the picture ſuch, as ſhall excite in the mind of the obſerver the ſame ſenſations as would ariſe therein, by ſeeing the real objects themſelves, which is not indeed to be expected barely from geometrical inveſtigations; recourſe muſt be had to the painter's ſkill in the art of colouring, for rendering the illuſions thus truly perfect: but, nevertheleſs, it is of the utmoſt importance to determine rightly the poſitions which the ſeveral lines and points, terminating the original objects, muſt have on the picture, ſo as to enable them, when duly coloured, to produce the effect wiſhed for by the artiſt; whence the arts of perſpective and colouring ſhould go hand in hand, and afford reciprocal aſſiſtance to each other.

From whatever point it be that a picture is viewed, the repreſentations will appear as the images of objects correſponding to that poſition; whence the ſame repreſentations may be made to allude, by changing the place of the eye, to different original objects, conſequently can not be a true exhibition of the objects there depicted, whoſe real forms, magnitudes and poſitions ought to be invariably preſerved, as ſeen from one certain point, that is to ſay, the true point of ſight; now every different poſition of the eye produces correſponding altera-

tions in the apparent places and other affections of the originals, and diſpoſes the mind to judge of them differently from what they really are in their natural ſtate; for different poſitions of the eye, taken in the line which is the parallel of the eye, belonging to the plane containing the original objects, will produce alterations of one ſort, while others, taken in the eye's director, occaſion changes of another kind; and the placing the eye nearer to or farther from the picture than it ought to be, cauſes variations different from either, as we ſhall now endeavour to make appear, by a few general obſervations derived from what has been before delivered.

And firſt, let us ſuppoſe the eye of the ſpectator to be placed ſomewhere in the parallel of the eye out of the true point of ſight, then will the center of the vaniſhing line appear to be as far diſtant from the true center as is the ſuppoſed place of the eye from the true place, and on the ſame ſide thereof; but the vaniſhing line of the plane which belongs to that parallel of the eye, as alſo the height of the eye, will remain without alteration, and thoſe dimenſions of objects, which are parallel to the picture, will preſerve their true magnitudes and poſitions: if the original plane be horizontal, it will ſtill appear ſo; the elevations of objects, ſituated thereon, will continue to appear perpendicular to it, and the ſame judgment will be paſſed on all lines in the ichnography which are parallel to the picture, as would be done from the true point of ſight; but the angles, formed by lines which incline, the planes raiſed upon them, and the apparent meaſures of their ſeveral parts, will be different from what they really ſhould be. For the true center of the vaniſhing line is the vaniſhing point of the images of all lines in the ichnography which are perpendicular to the interſecting line, and when the apparent place thereof is changed, that true center then becomes the vaniſhing point of ſuch original lines as incline to the line of ſtation,

or line which determines the poſition of the ichnography, ſo as to form therewith an angle, whoſe tangent is the diſtance between the real and apparent centers, the diſtance of the eye being made radius, and a correſpondent change will enſue in the appearance of all other original lines, whoſe images tend to any other vaniſhing points, and conſequently in the apparent inclinations of any elevated planes, whoſe vaniſhing lines paſs through thoſe points.

PLATE IX. Fig. 10.

Suppoſe now the eye to be placed in e, and draw eV perpendicular to the vaniſhing line H L, then will V be the apparent center; for, in whatever point the eye be placed, a line drawn from thence perpendicular to the picture, the point wherein that line cuts it, will be judged to be its center; and x C, the whole image of an original line, perpendicular to the interſecting line, will appear as inclining to the line of ſtation, which is parallel to E C, under the angle V e C; now, if e V or E C be aſſumed the radius, then is V C the tangent of the ſaid angle V e C. And if e V or E C were the vaniſhing line of a plane paſſing through x C perpendicular to the picture and original plane, it will appear, from the ſtation e, as a plane perpendicular indeed to the original plane, but it will ſeem as inclining to the picture in the angle e C V: hence it is that a plane, which would appear perpendicular to the picture when ſeen by an eye at E, will, as the eye is removed in the line E X, ſeem to incline more and more towards the picture, and always the contrary way to that in which the eye is tranſlated from one place to the other; for inſtance, the plane ſeems to incline towards Z, as the eye moves from E towards X, and when the eye moves from E towards Z, then it ſeems to incline in like manner towards X.

If any two vanishing points, H and h, be taken, and there be drawn from the true point of sight the lines E H, E h, they will subtend the true visual angle under which those two points will appear; also drawing the lines e H, e h, they will subtend the angle under which the same points will appear from the place or station e, and consequently all original lines tending to those points will appear at e, under the angle H e h, which will be equal, greater or less than the true angle, according where the points H and h happen to meet the vanishing line H L, with respect to the real and apparent centers C and V.

For if, upon the line H h, such a segment of a circle be described on the vanishing plane, which, passing through E, contains the true visual angle H E h, (33. 3.) it is plain that, in whatever point of its circumference the eye be placed, the angle subtended by lines drawn from thence to the points H and h, will appear the same; therefore if the eye be at q, the point where it cuts the parallel of the eye, the apparent angle H q h will be equal to the true angle; but, if the eye be in or out of the circle bounded thereby, the apparent angle will be accordingly greater or less than the true angle H E h.

Now, as the line x C appears at the point e to incline to the picture, the line e C becomes that which apparently produces its vanishing point; and therefore, if x C be any how divided, suppose in the points n, e, &c. the apparent measures of the parts x n, n e, &c. will be increased or diminished in the same proportion which the apparent line e C has to the true line E C, which really produces the vanishing point of that line; so that, instead of representing their true measures x I, I J, from the place e, the mind will judge them as being equal to x w, w l. And seeing that the apparent line, producing the vanishing point, may be equal, greater or less than the true one, according to the position which any vanishing point in the

line H L has to the apparent center V, excepting only the point C, for the apparent line belonging thereto can never be leſs than E C, it follows, that the images of the parts of any inclining original line may accordingly repreſent parts equal, greater or leſs than their true originals; but, notwithſtanding this, the perpendicular diſtance between the picture and the points n, c, &c. will appear true, ſo long as the eye continues in its parallel X Z. The ſame holds good alſo with reſpect to any elevated plane, whoſe vaniſhing line is E C, or a line parallel thereto; for, although its center is not varied while the eye moves in the line X Z, yet the line, which apparently determines its vaniſhing point, becomes equal to e C, and as that is greater than the true one, a correſponding effect will be thereby wrought on the apparent lines belonging to all other vaniſhing points in that line, as alſo on the apparent angles and diſtances from the interſecting line of the originals of any points in that elevated plane, though not on their perpendicular diſtances from the picture.

The eye being placed in any point of the director of any original plane, thoſe dimenſions of objects, which are parallel to the picture, appear unvaried both with reſpect to their magnitude and diſtance as above-mentioned; but the apparent place of the horizon will be changed, and ſeem as being above or below the true horizontal line, according as the eye is higher or lower than the true point of ſight; the original plane will accordingly appear as raiſed above or ſunk below the horizon, and the perpendicular ſupports of all original points appear as oblique ſupports; alſo the apparent line, producing the vaniſhing point, will be increaſed, by altering the apparent place of the center of the picture, the originals of the parts of all inclining original lines will be judged proportionably greater than they really are, and the objects will thereby ſeem to occupy more ſpace in depth, on this inclining plane, than they actually do, though their perpen-

dicular distances from the picture are not affected. Now, seeing the eye is supposed always in the same director, the vertical line of the original plane will remain unaltered, and consequently all elevated planes will appear inclining to the picture under the same angles as they would do from the true point of sight, but will not appear perpendicular to the original plane, but as the planes of the oblique seats of lines on that plane, and those lines in the planes of elevation which should appear parallel to the horizon, such as the ranges of windows, cornices, or other members in buildings of architecture, by the apparent change of their centers, will appear elevated or depressed above or below the horizon, according as the height of the eye is greater or less than that of the true point of sight. Also the apparent angles, formed by inclining lines in those planes, and the apparent magnitudes of their parts, will be in like manner affected from the apparent changes in the centers, made by varying the lines which apparently produce the vanishing points thereof. But so long as the eye continues in the same director, the perpendicular distances betwen the picture and all the points in those planes continue to appear as they should do, by reason the director is, in this case, the parallel of the eye with respect to the said elevated planes.

But, if the eye be placed any where in the line producing the vanishing point of any original line, farther from or nearer to the picture than the true point of sight, the center of the picture, horizontal and vertical lines will retain their true appearance, and all planes of elevation will continue to appear perpendicular to the original plane; but the distance of the eye, being thus varied, will have an effect upon the apparent magnitudes of the parts of all inclining original lines, and consequently on the apparent distances of those parts from the picture, and likewise on the apparent angles subtended by lines joining the vanishing points of lines in that plane,

for they will appear greater or less than they truly ought, according as the distance of the eye is lessened or increased; also the apparent inclinations of all elevated planes, and lines therein, with respect to each other and the picture, will be affected, except only the right angle contained by the parallels and perpendiculars to the intersecting lines of the several planes, for they will all continue to appear perpendicular to each other, wherever the eye be placed in the line here mentioned, because the center of the picture continues the same, the centers of all vanishing lines in the picture will remain unvaried; and the magnitudes of the parts of those parallels, and of all other lines which measure the dimensions of objects parallel to the picture, will be judged the same.

PLATE VIII. Fig. 9.

Thus, suppose S H to be the picture, C its center, O C its distance, O being now taken for the true point of sight, if that distance be increased as to E, the apparent line above-mentioned will then be E C, and the apparent angle L E H, subtended by the line joining the vanishing points L and H, will be less than the true angle L O H, and would be more so, were the eye in some other point of that line between O and C, and the rest, it is plain, is as above declared.

Now, let Y E be the vertical plane, C the center of the picture, O C its true distance, D F the line of station; let A B be an original line in that plane perpendicular to the original plane, and parallel to the picture, whose image, seen from O, the true point of sight, is k y or q y. Let now the eye be supposed in E, then will K U, which is the apparent original of the image k y, although it appears farther distant from the picture than its true original A B, be judged,

by the mind of the ſpectator, to be of the ſame ſize as A B; that is, A B and K U will be equal to each other.

For, in the ſimilar triangles E O k, k A K,

$$Ek : kK :: Ok : kA,$$

by compoſition $Ek + kK = EK : Ek :: Ok + kA = OA : Ok.$

But, in the ſimilar triangles E k y, E K U,

$$EK : Ek :: KU : ky,$$

and, in the ſimilar triangles O k y, O A B,

$$OA : Ok :: AB : ky;$$

therefore, by a parity of reaſoning,

$$KU : ky :: AB : ky,$$

conſequently $KU = AB$.

If we ſuppoſe E the true point of ſight, and O the point aſſumed for the place of the eye, the ſame reaſoning will ſhew, that the diſtance of the eye being leſſened, the original of k y will be judged nearer the picture than it really is, but ſtill of the ſame magnitude.

Now, ſeeing it is as A B to k y, ſo are all other lines in a plane parallel to the picture paſſing through A B to their images ſeen from the ſame point O, and likewiſe as K U is to k y, ſo are all other lines in a plane parallel to the picture, paſſing through K U to their images ſeen from E; therefore, ſeeing A B and K U are equal, the originals of all lines in the picture, which meaſure the dimenſions of objects parallel to it, will be judged of the ſame length, let the eye be placed where it will in the line O C or E C.

What has been advanced relating to the ſeveral poſitions of the eye as being placed in its parallel, director, or as laſt-mentioned, is eaſily applied to any other poſition of the eye whatever; for, ſuppoſing it placed any where in the directing plane, the conſequences will be derived from what has been ſaid of the parallel and director,

and, if placed any where out of the directing plane, by taking in the above-mentioned considerations, the effects will readily appear.

Upon the whole, we may observe, that the ill effects produced on the picture, by its not being viewed from the true point of sight, are not so considerable by its being seen from a point that is too near or too far, so long as the eye remains on a level with the horizontal line, as are those which spring from a position of the eye that is higher or lower than it ought to be; and that a reasonable licence may be allowed for deviating from the true point of sight in viewing such pictures as are on a plane surface, without incurring the fault of any other inconsistencies or misrepresentations, but what may, in some sort, be corrected by the imagination of the spectator; but, as to those pictures which are formed on other kinds of surfaces, as domes, or arched roofs, or other irregular figures, it is there absolutely necessary to pay the strictest regard to the true point of sight; for any the least variation from it, occasions the most gross appearances, and makes the figures seem as being disjointed, broken or distorted, and the unity of the representation is thereby destroyed.

Having far exceeded the bounds first prescribed to this little Tract, we here conclude, reserving the doctrine of shadows, the method of applying perspective to scenographical representations, as now practised in painting the scenes of theatres, the manner of drawing anamorphoses or deformed appearances, &c. for a future disquisition.

FINIS.

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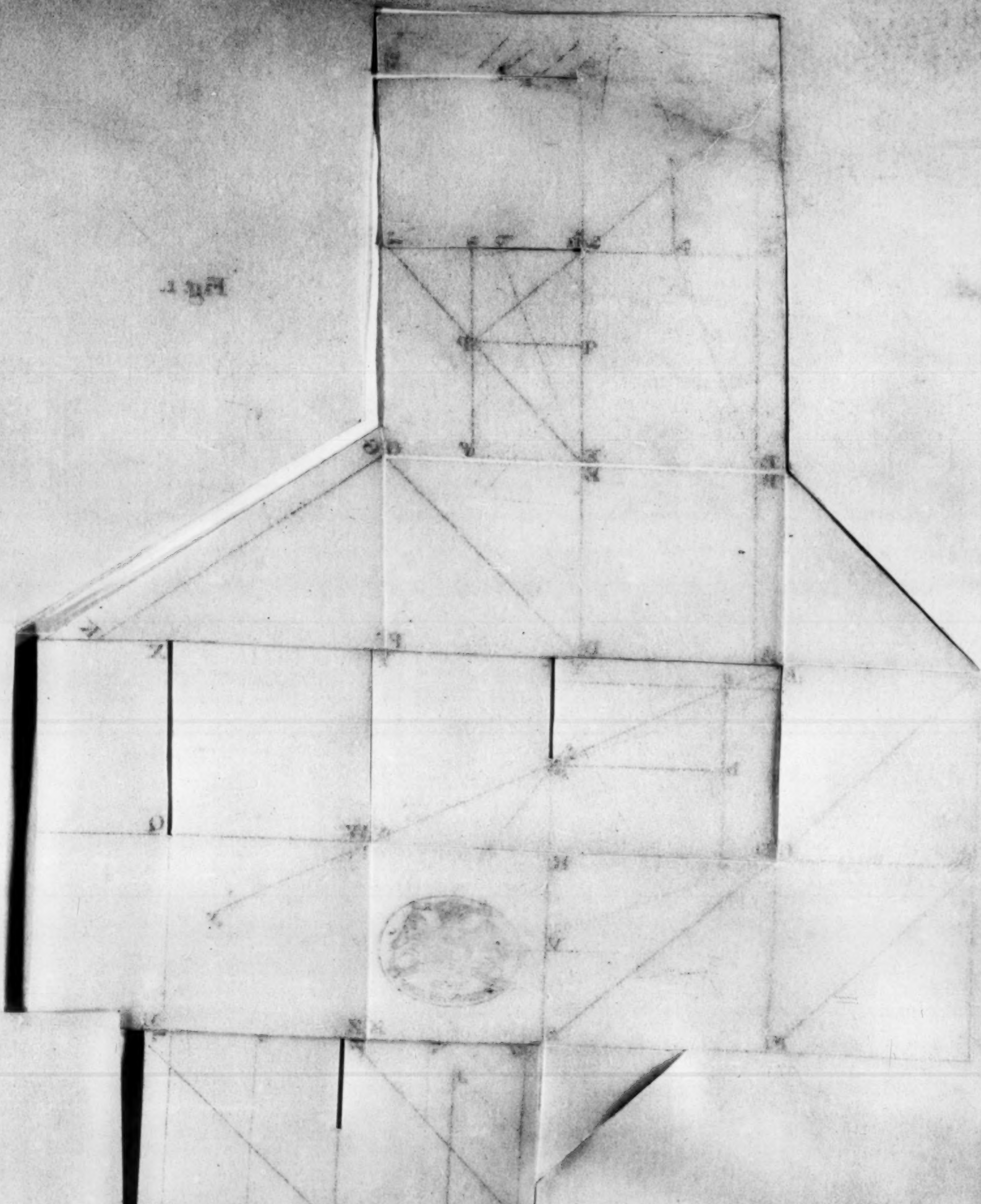

12

Plate I.

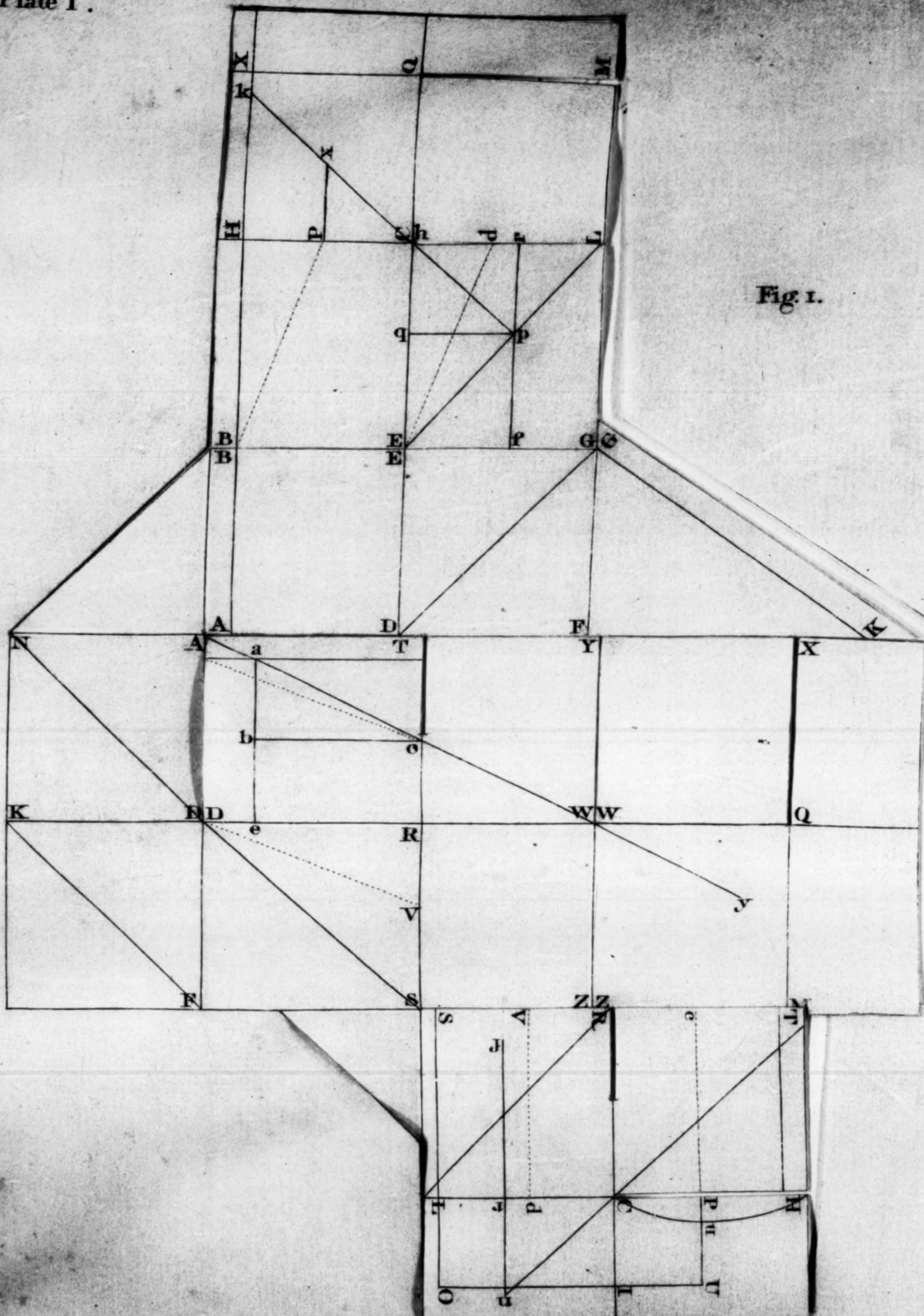

Fig. 1.

Plate I.

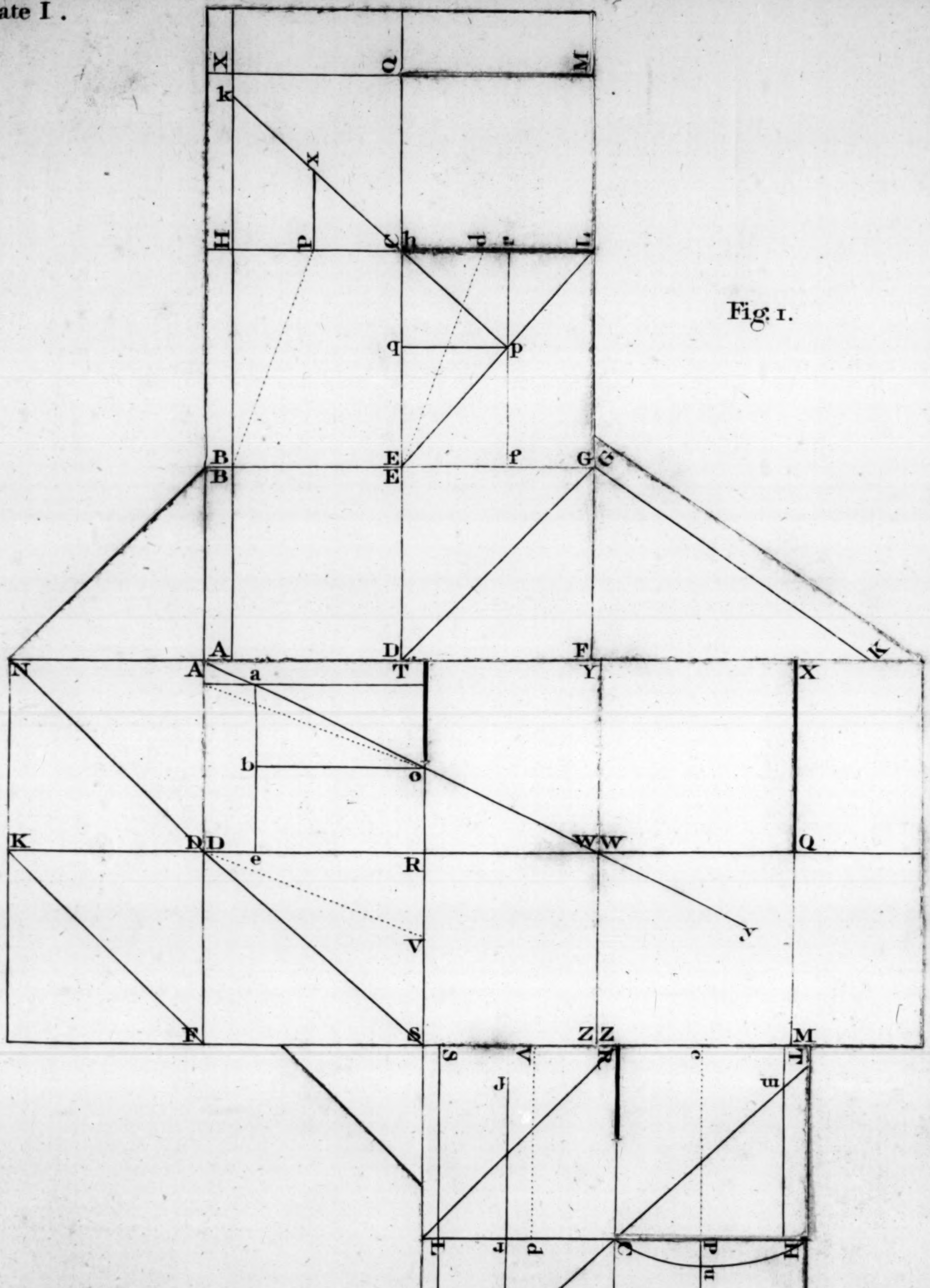
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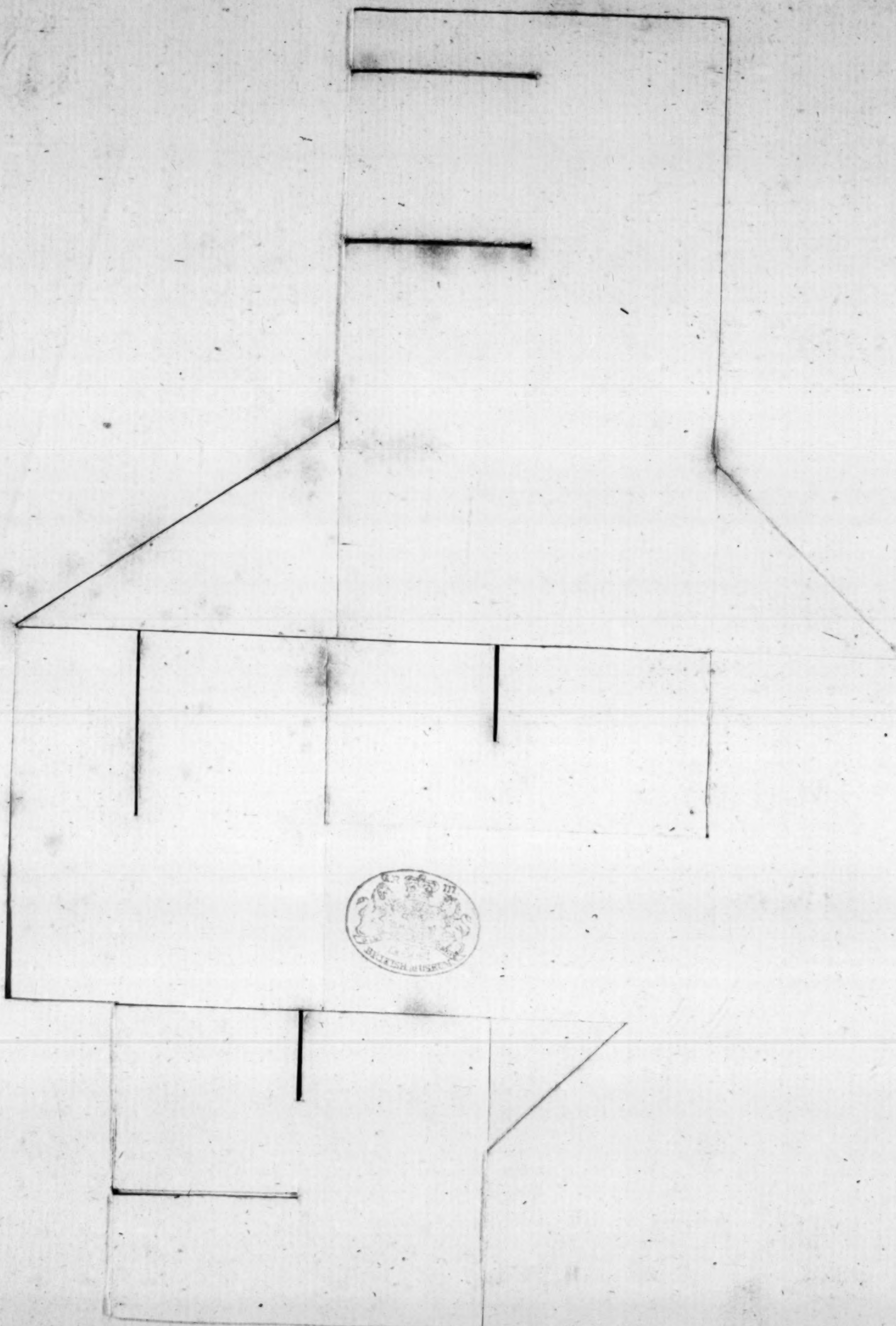
Fig. 1.



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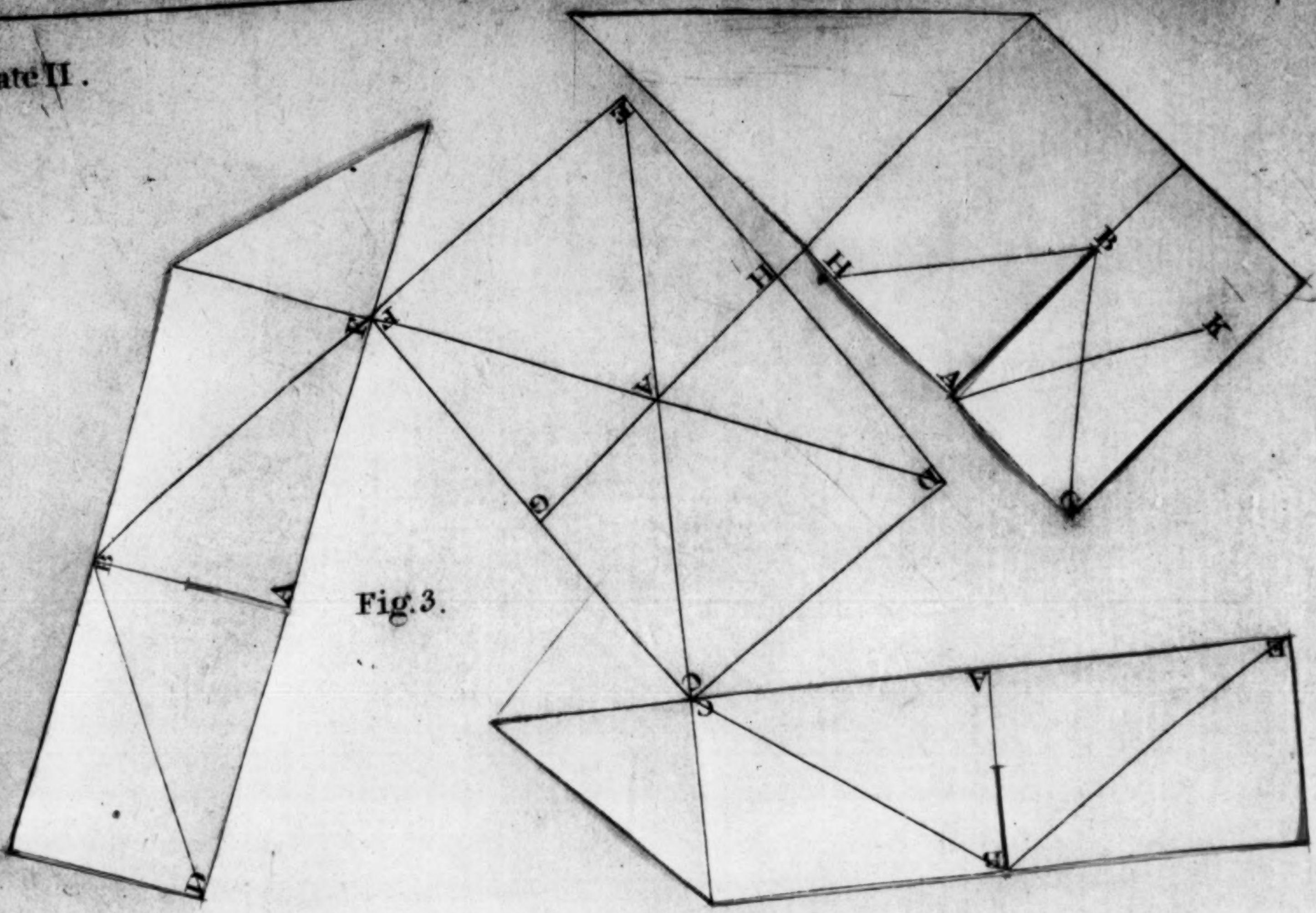

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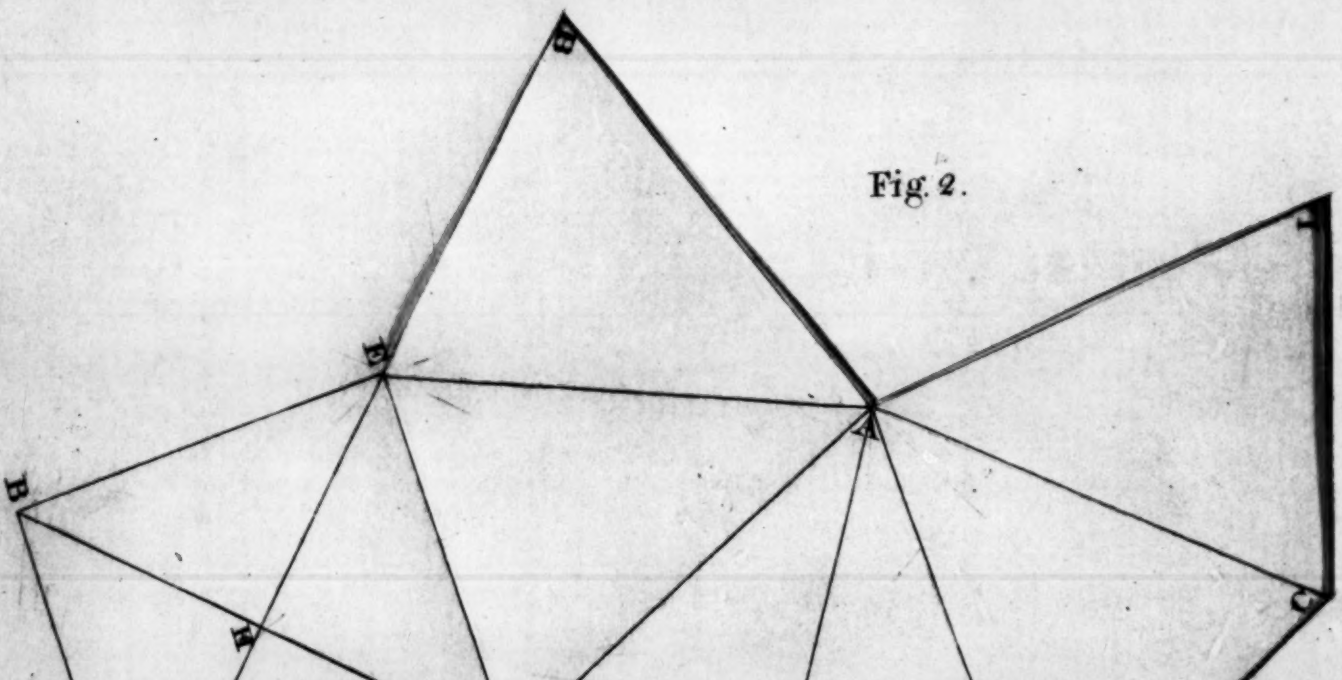

Fig. 2.

Plate II.

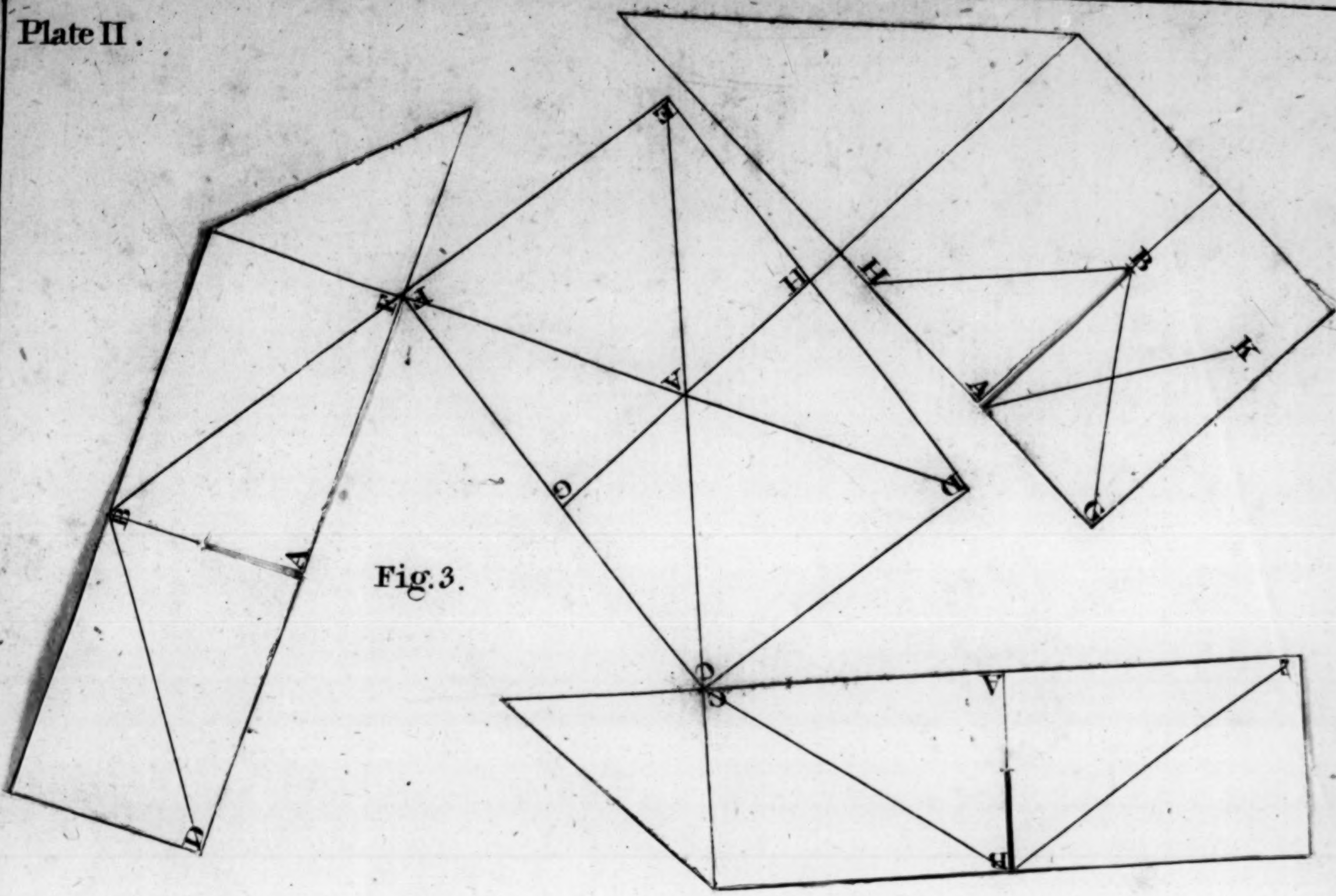

Fig. 3.

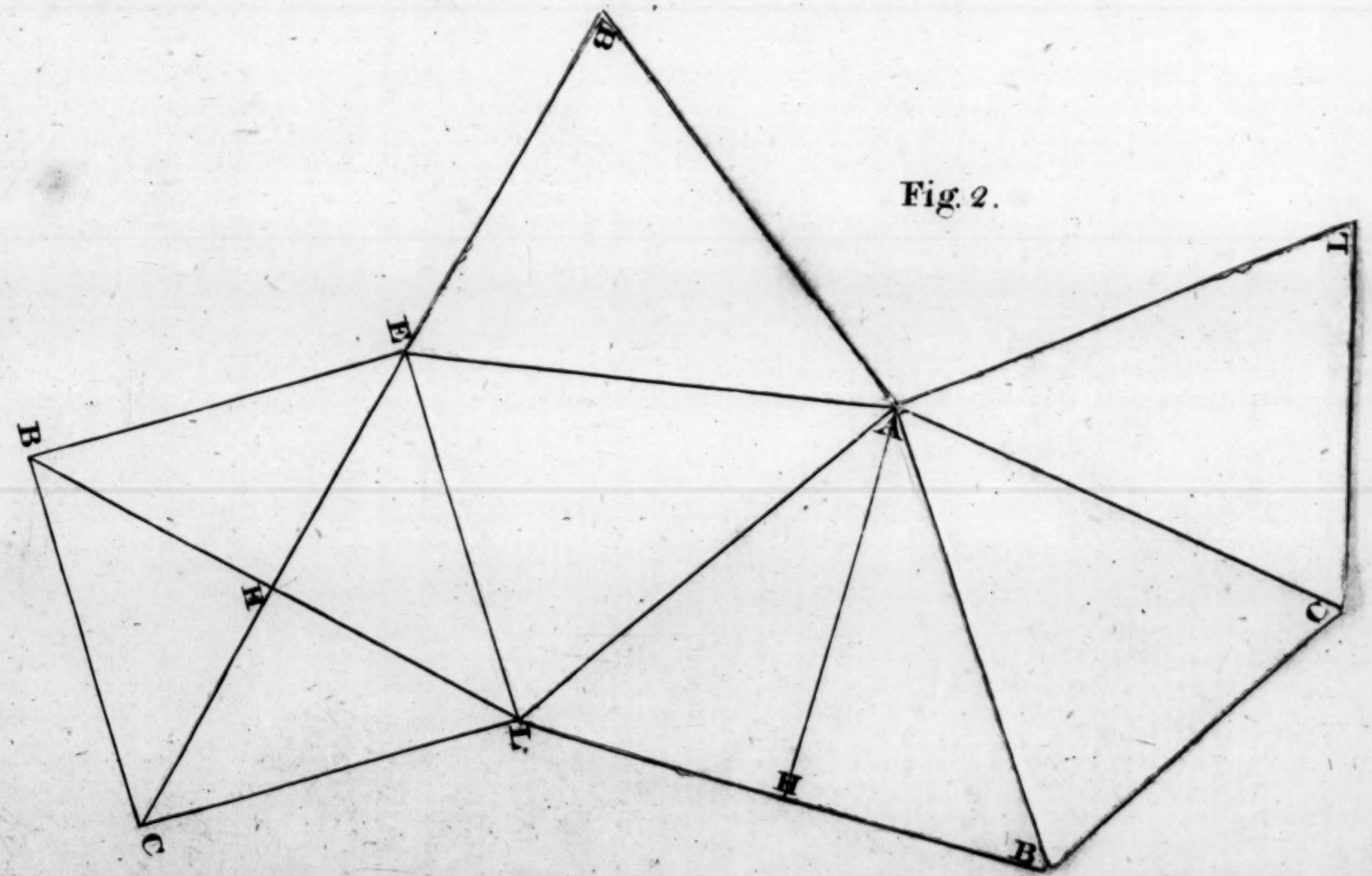

Fig. 2.

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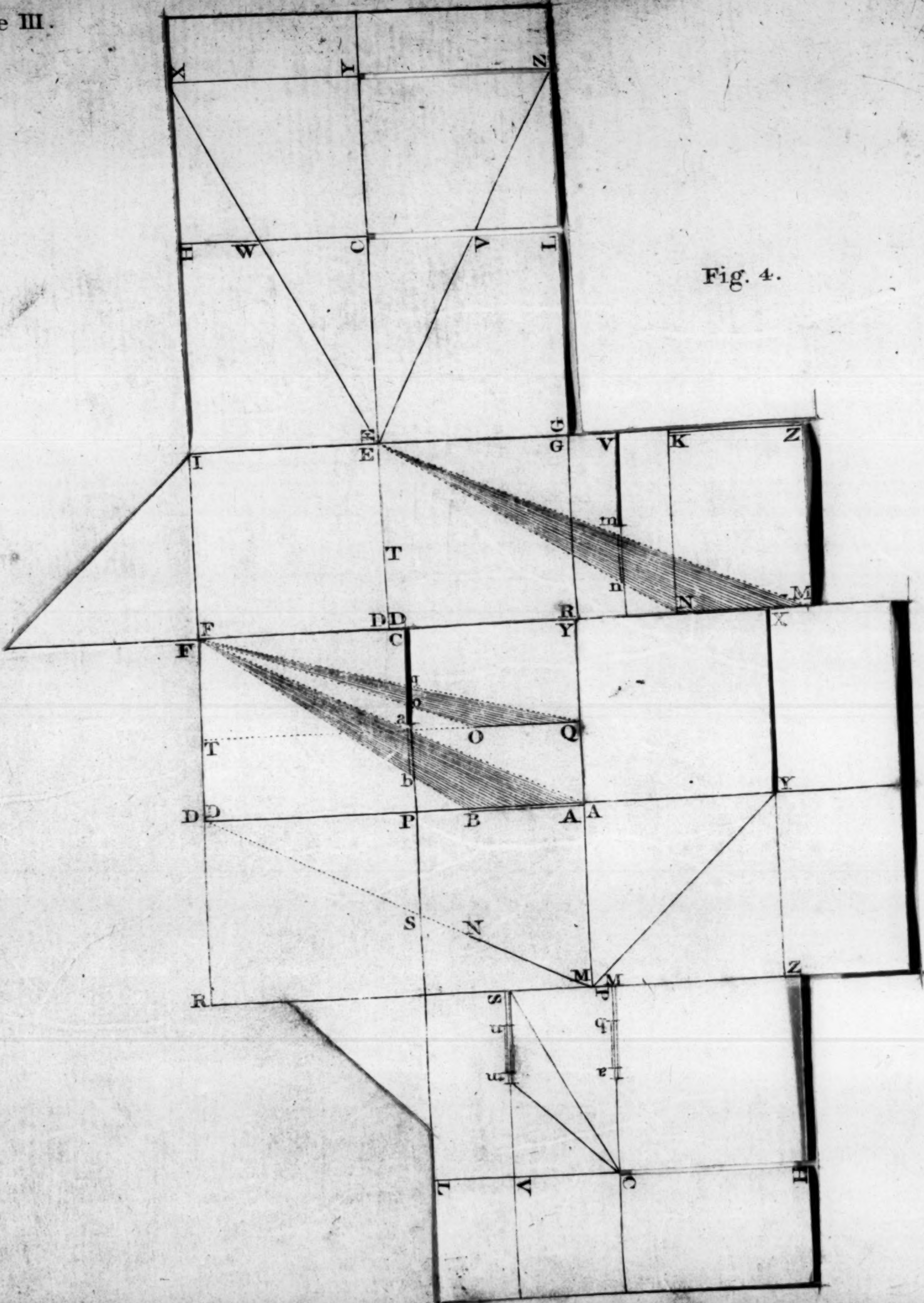

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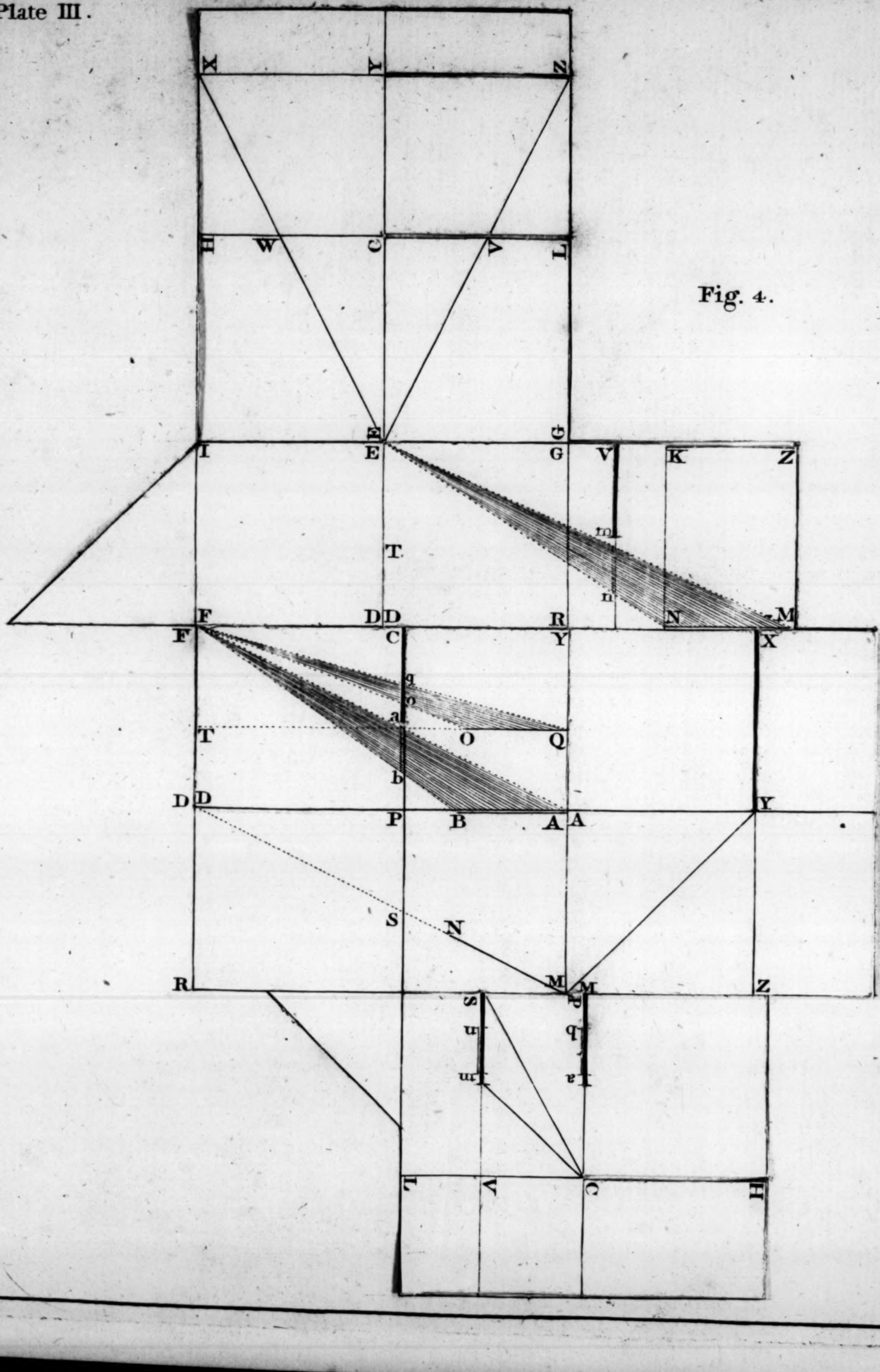

Fig. 4.

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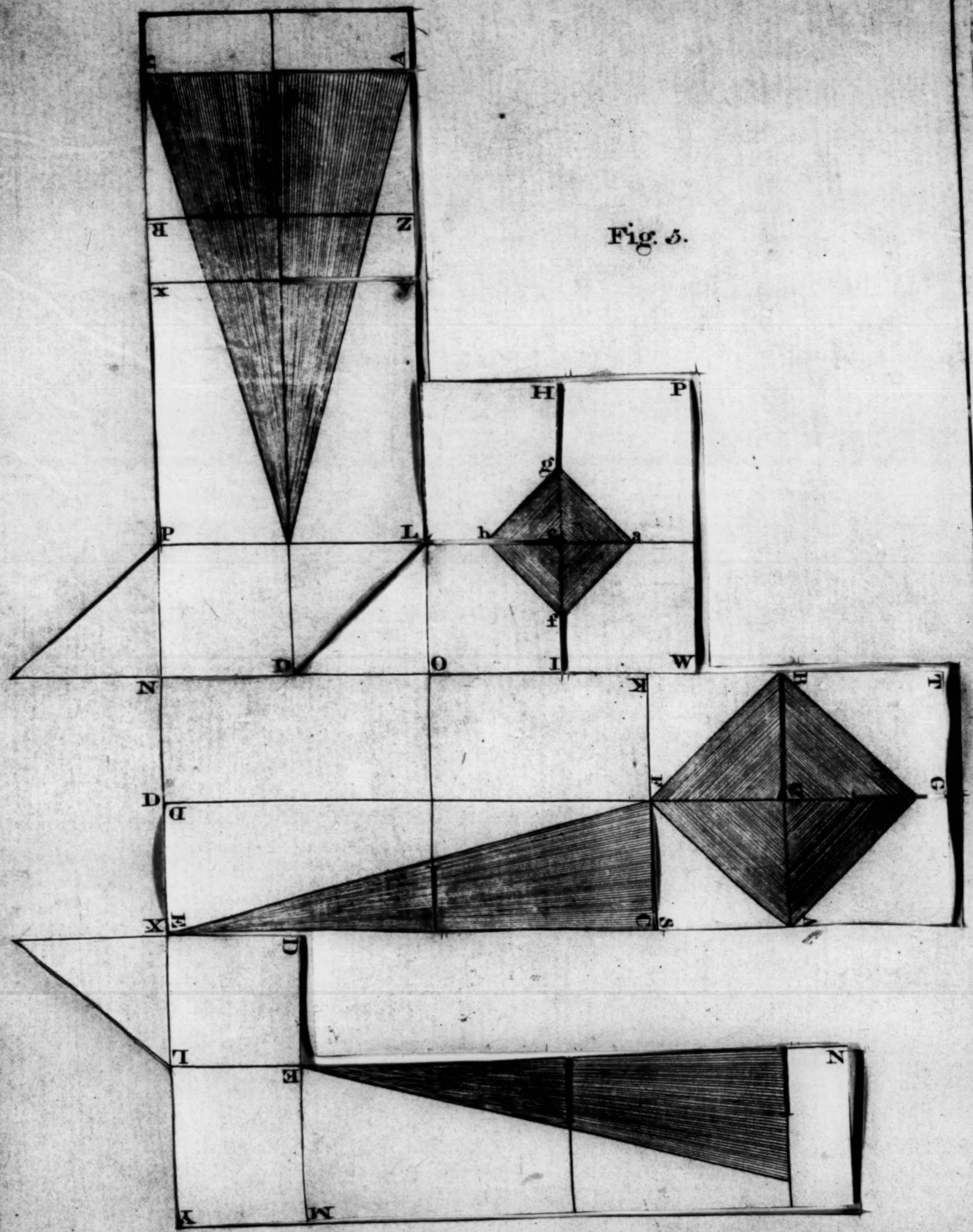

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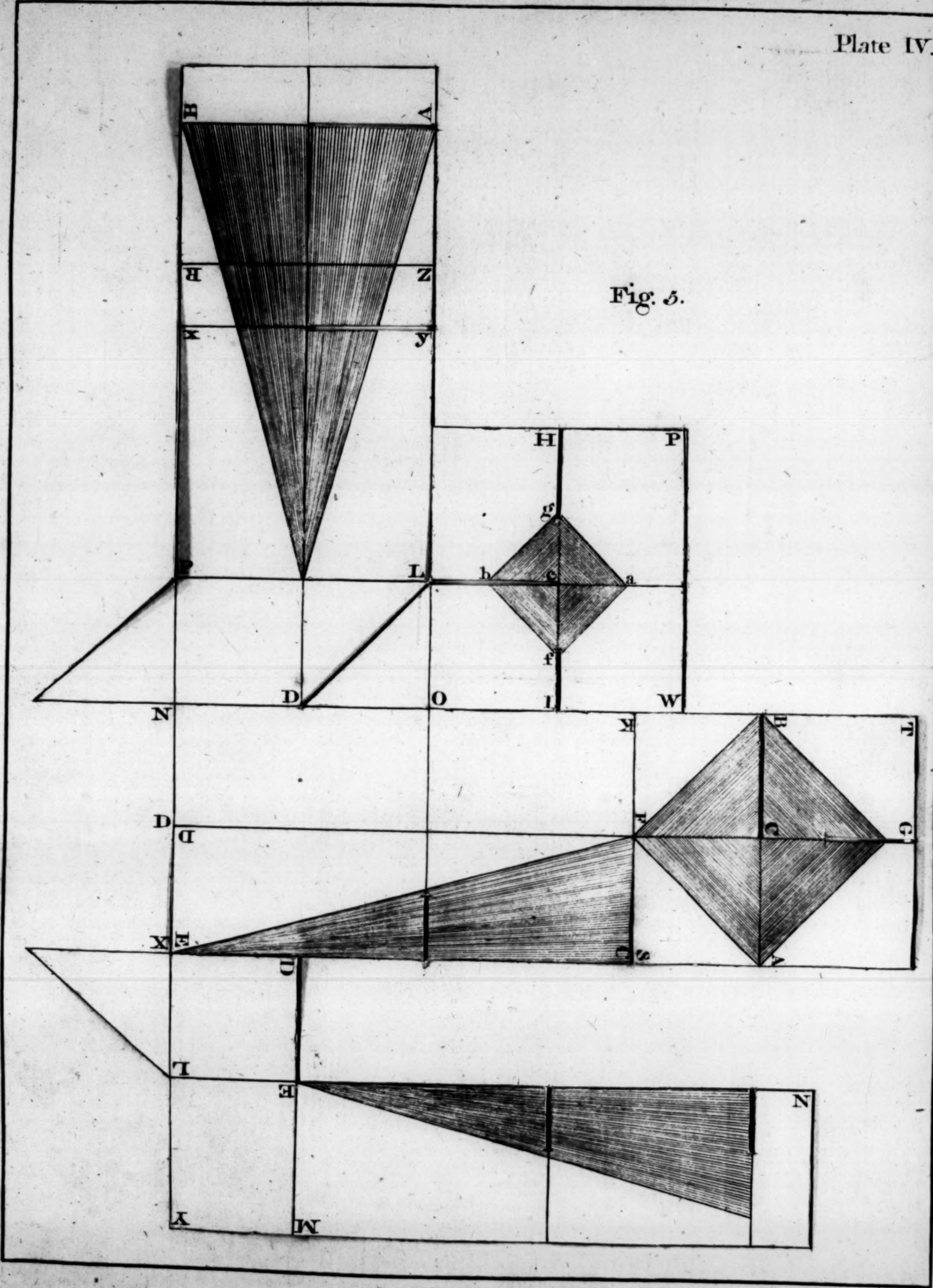


Plate V.

Fig. 6.

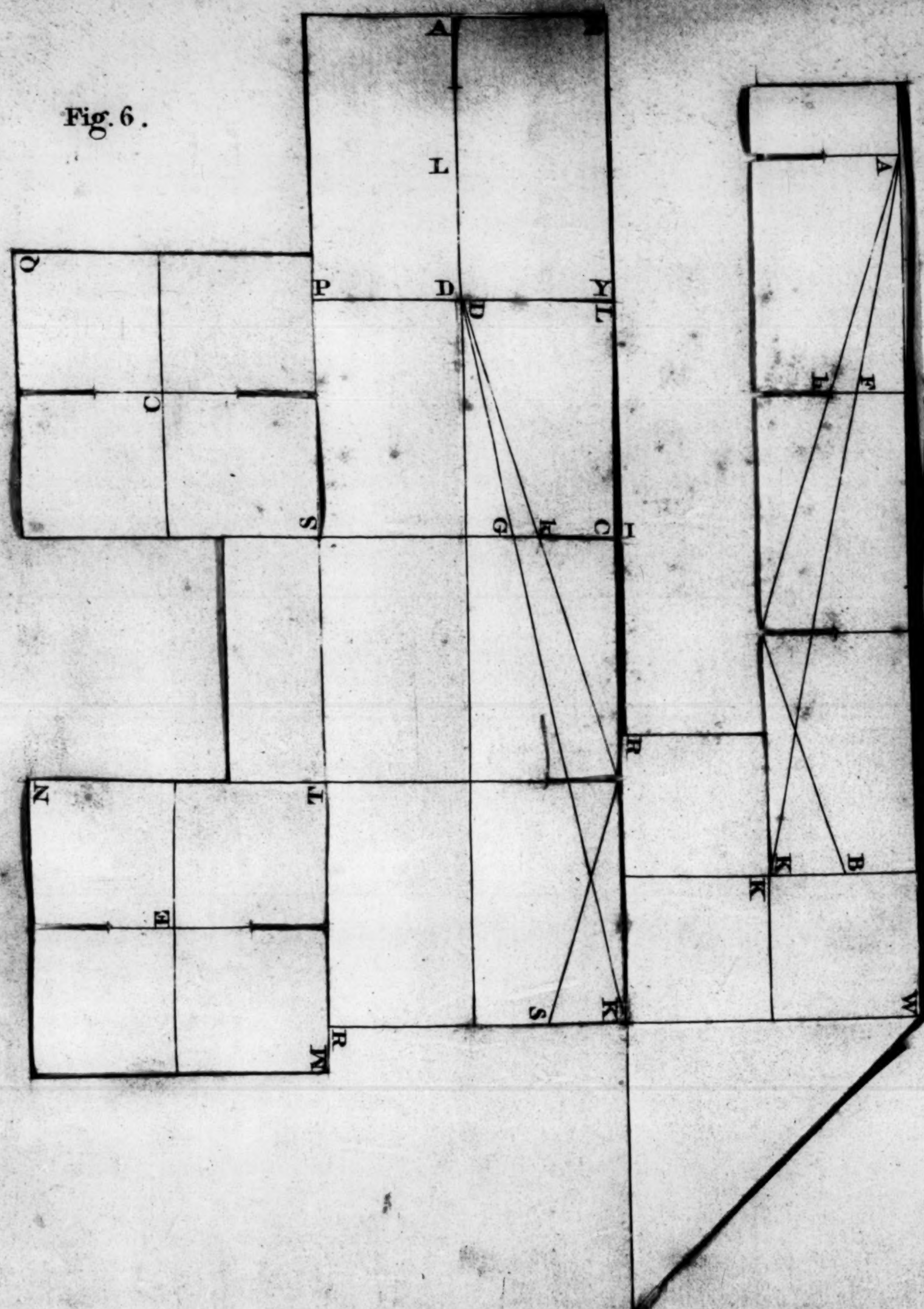
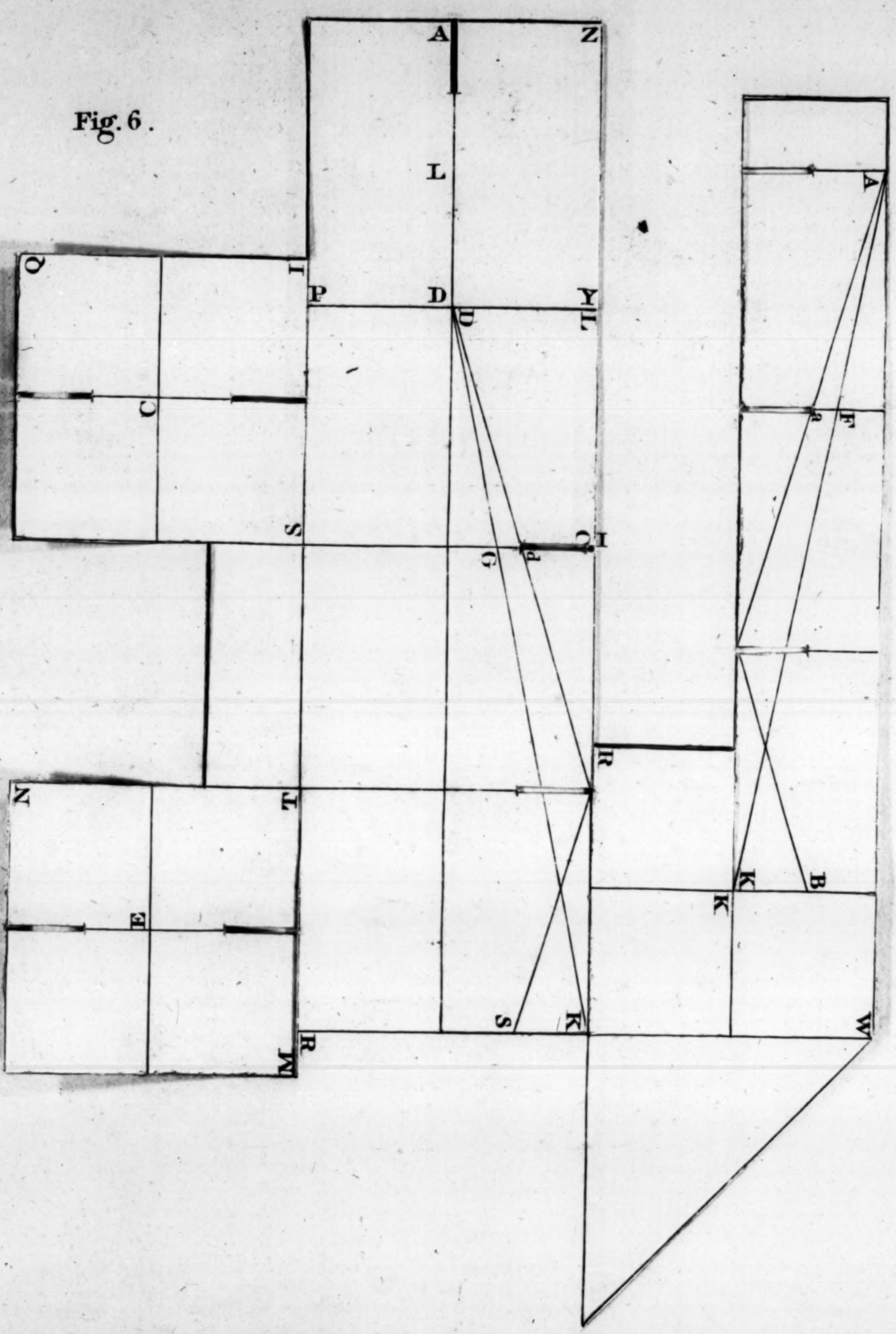


Plate V.

Fig. 6.



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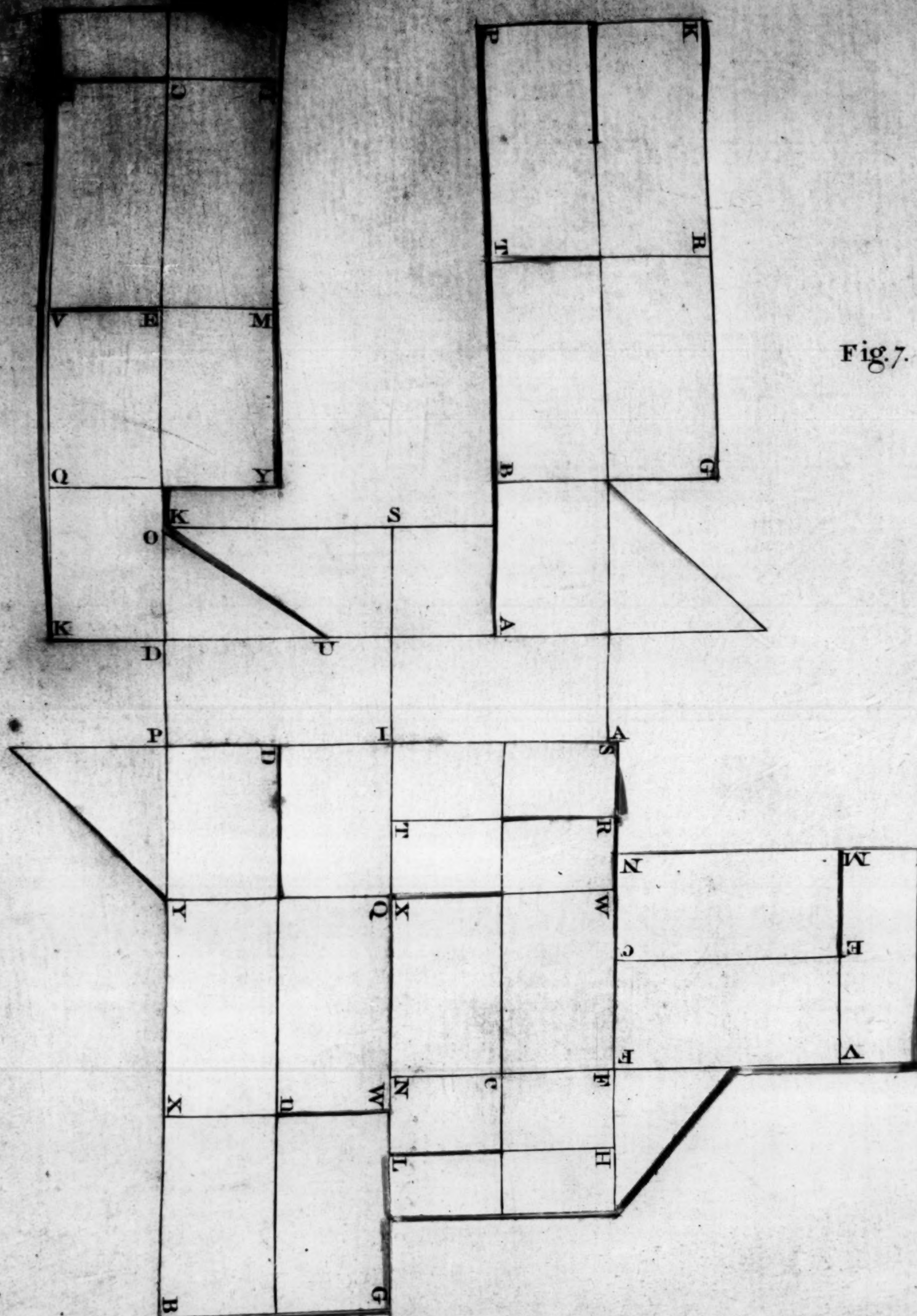


Fig. 7.

Plate VI

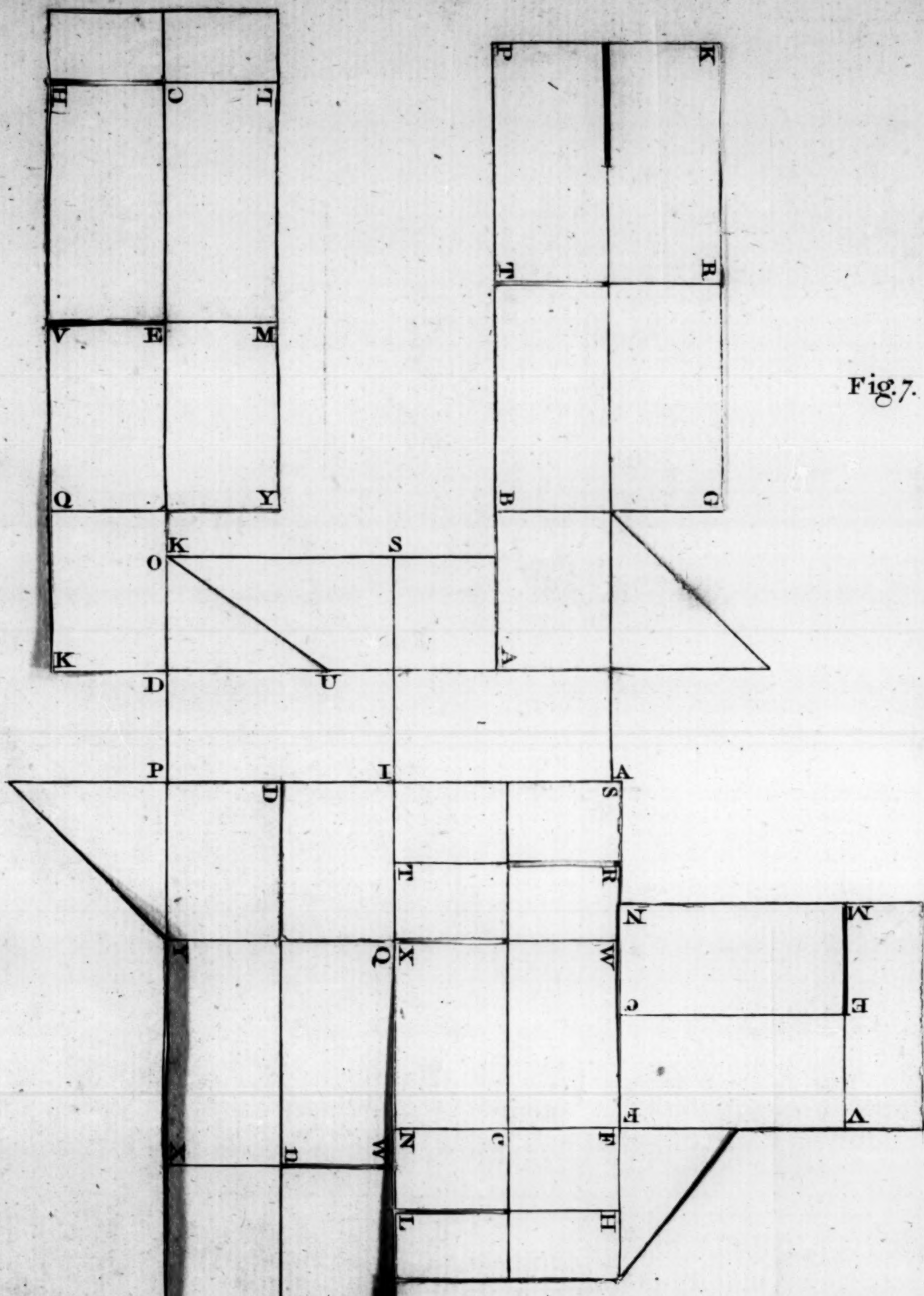


Fig. 7.

Plate VII

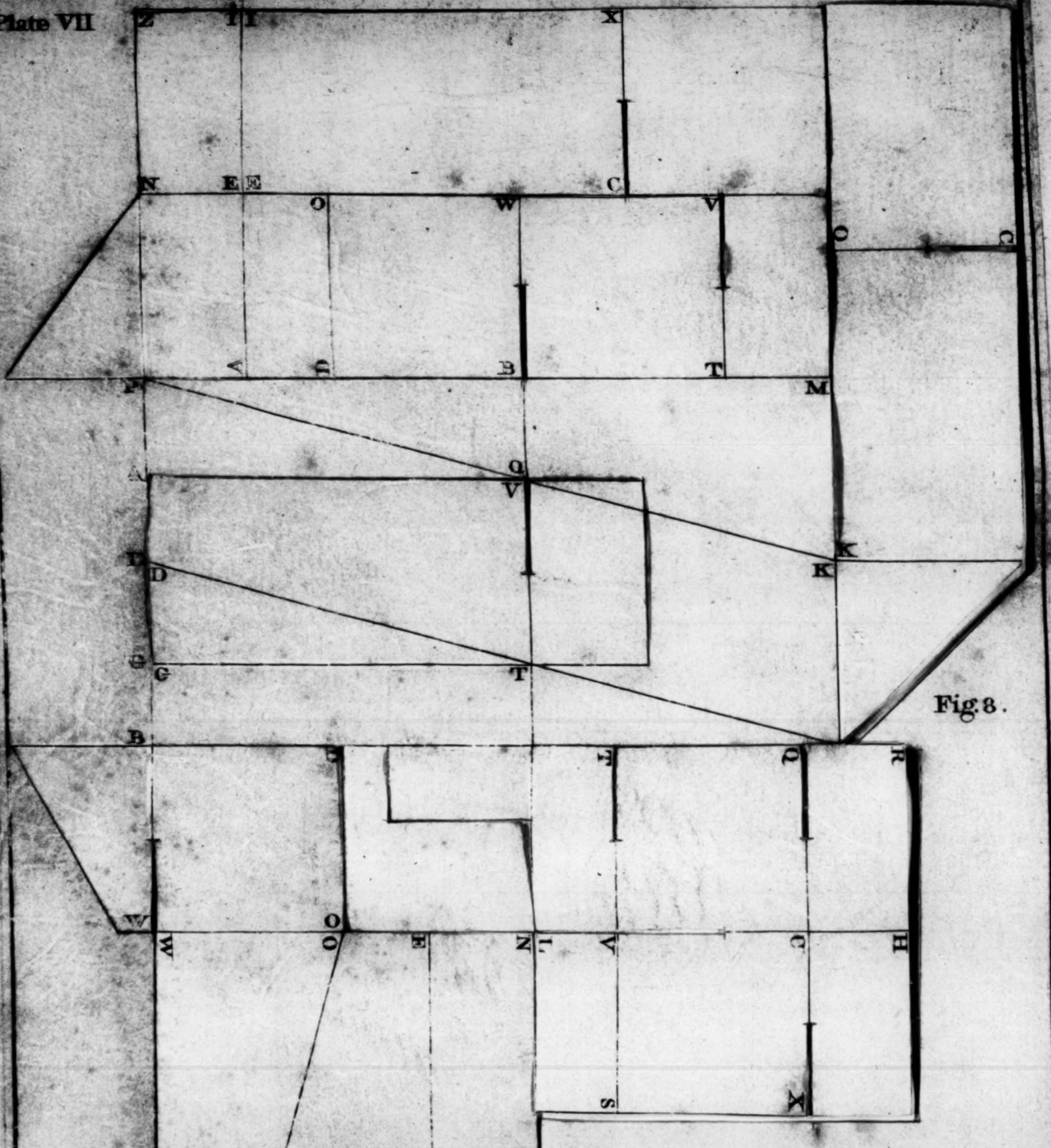
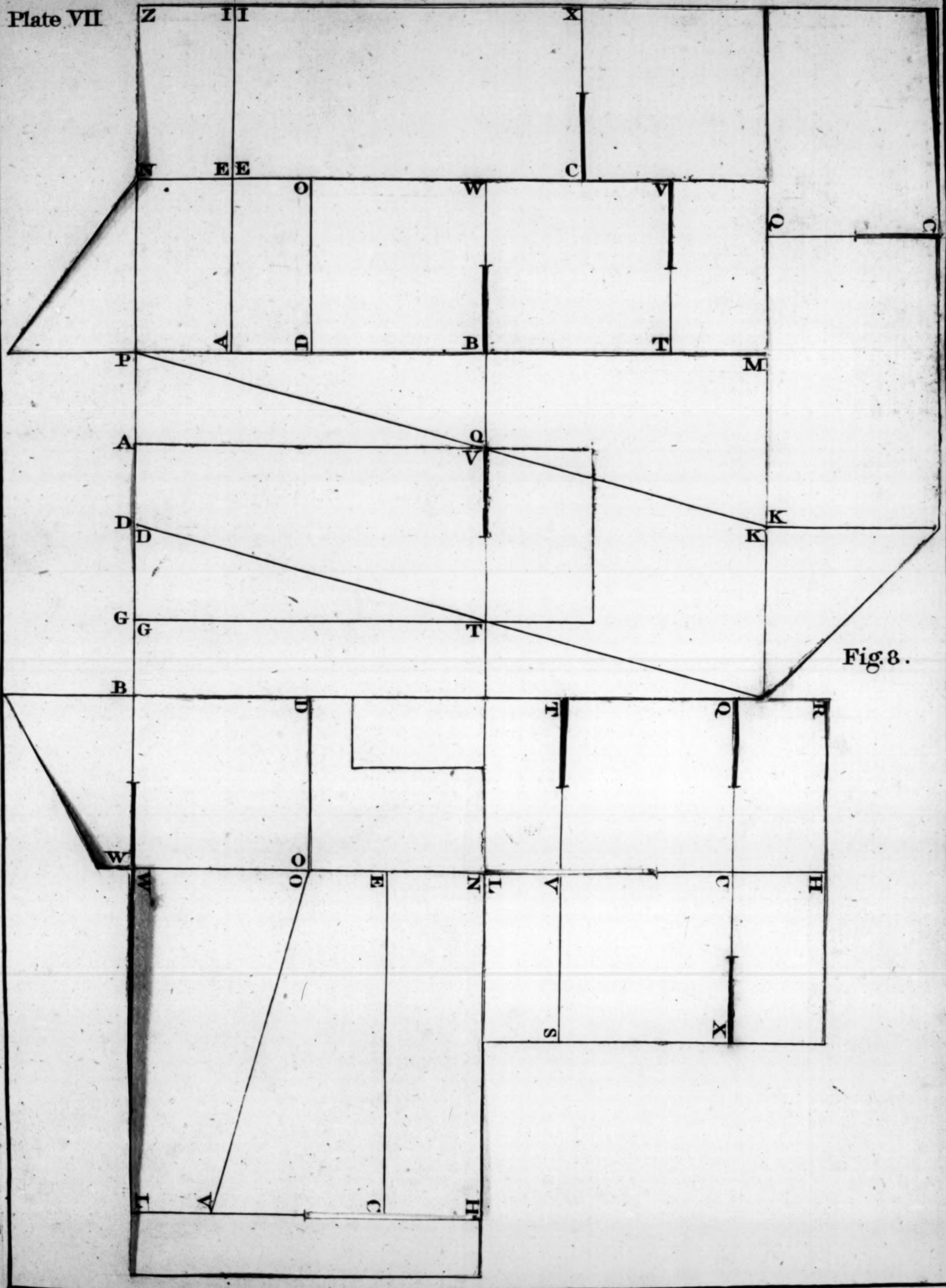


Fig. 8.

Plate VII



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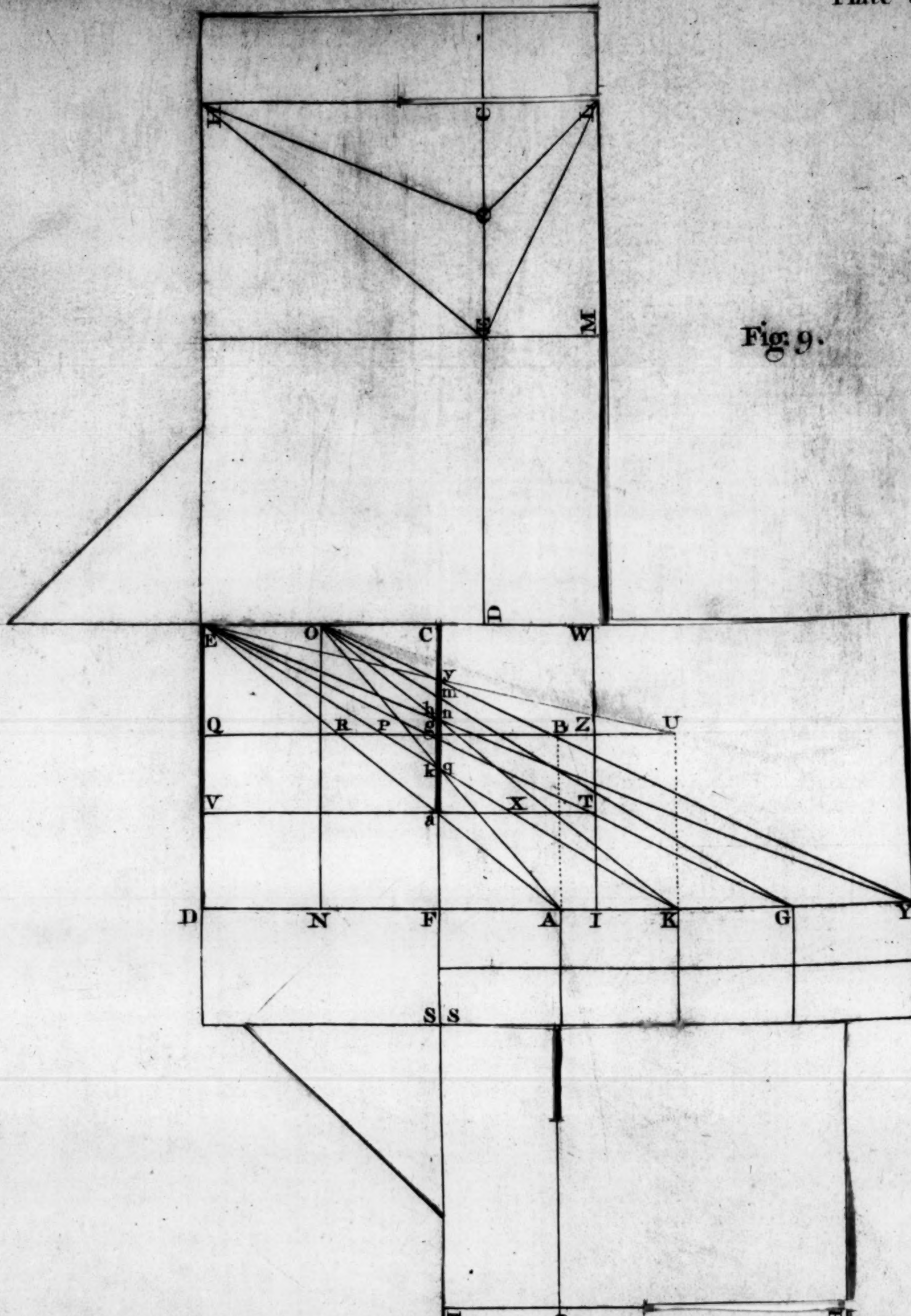

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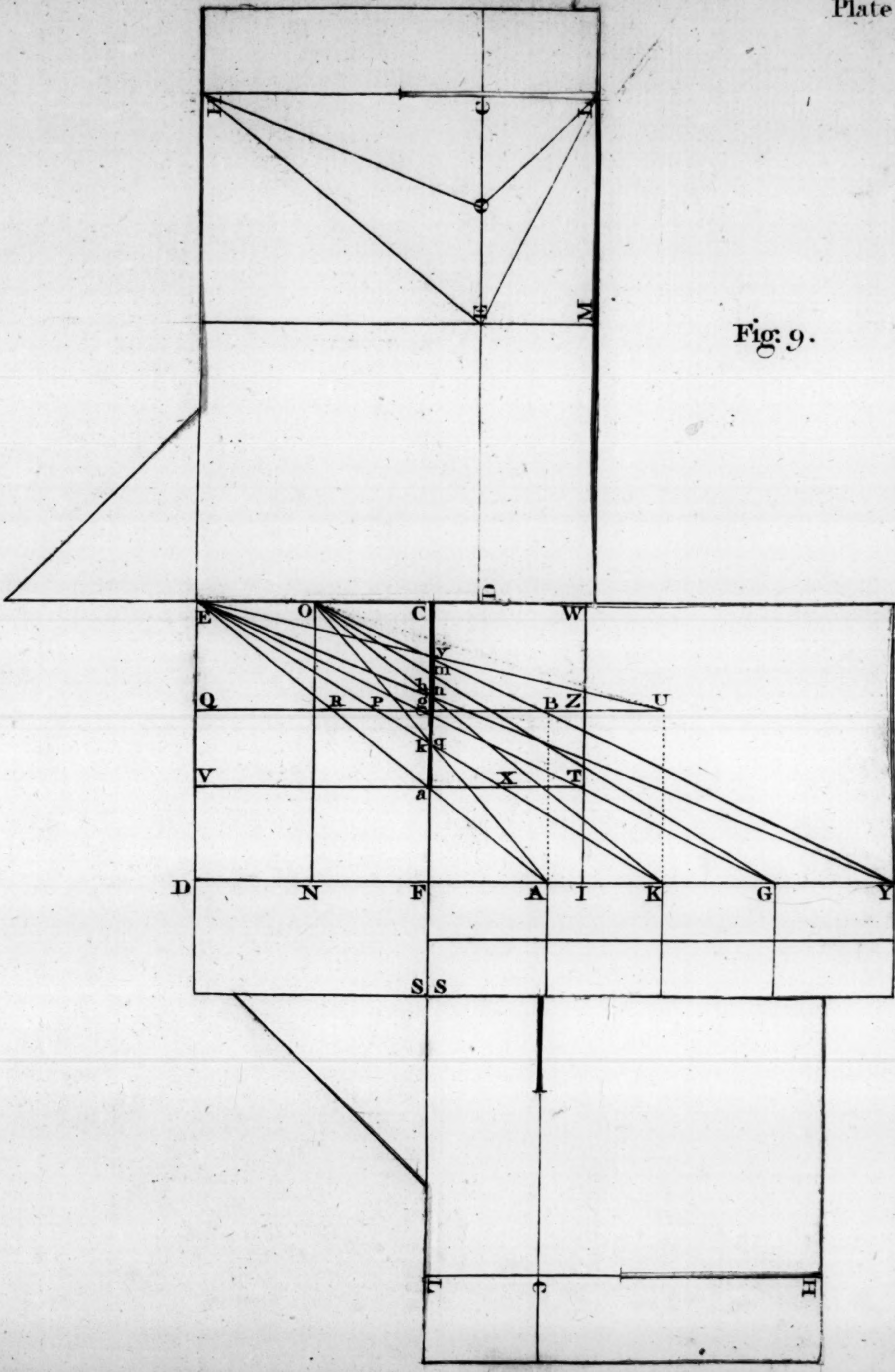


Fig: 9.

Fig.10.

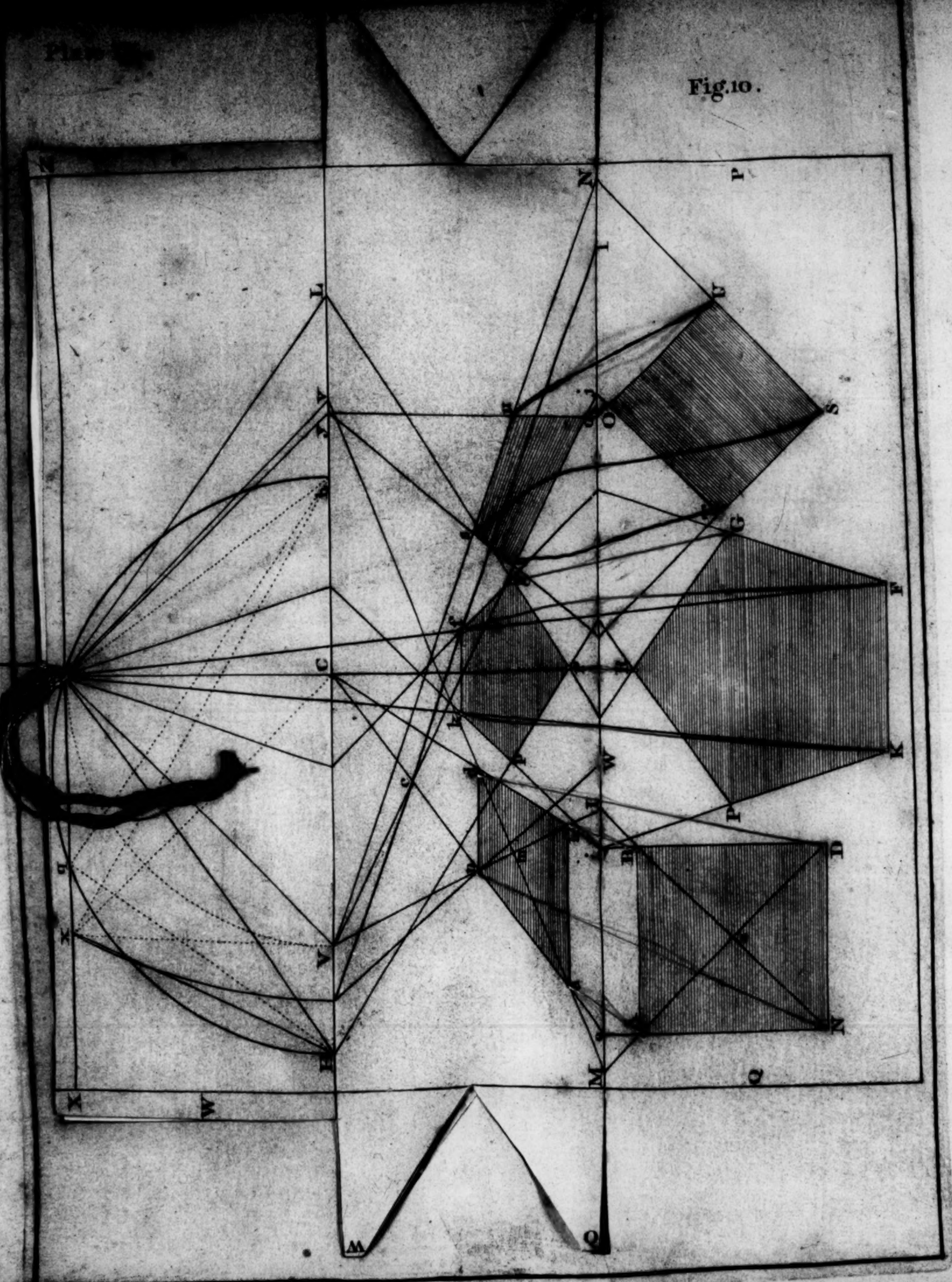
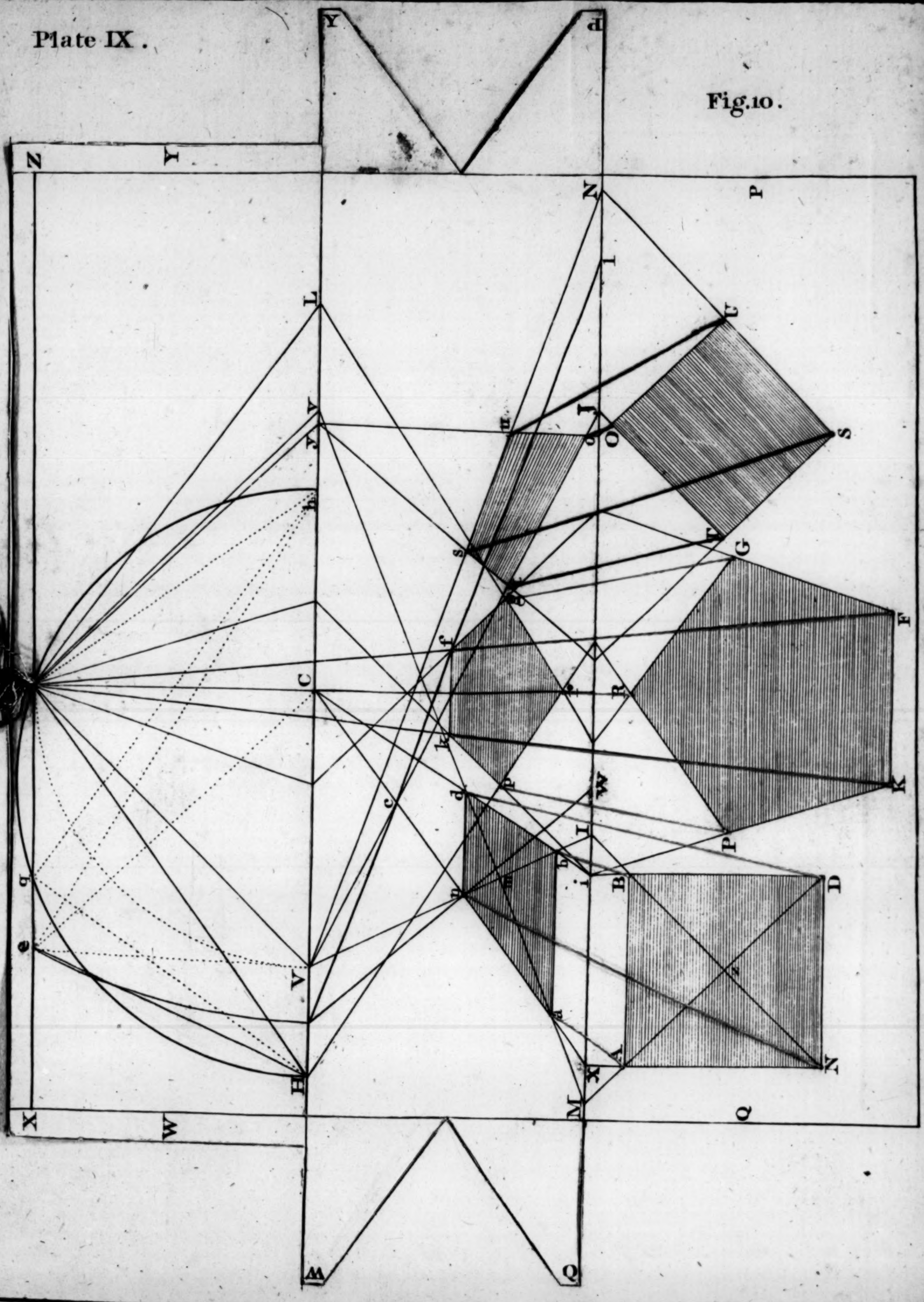


Plate IX.

Fig.10.



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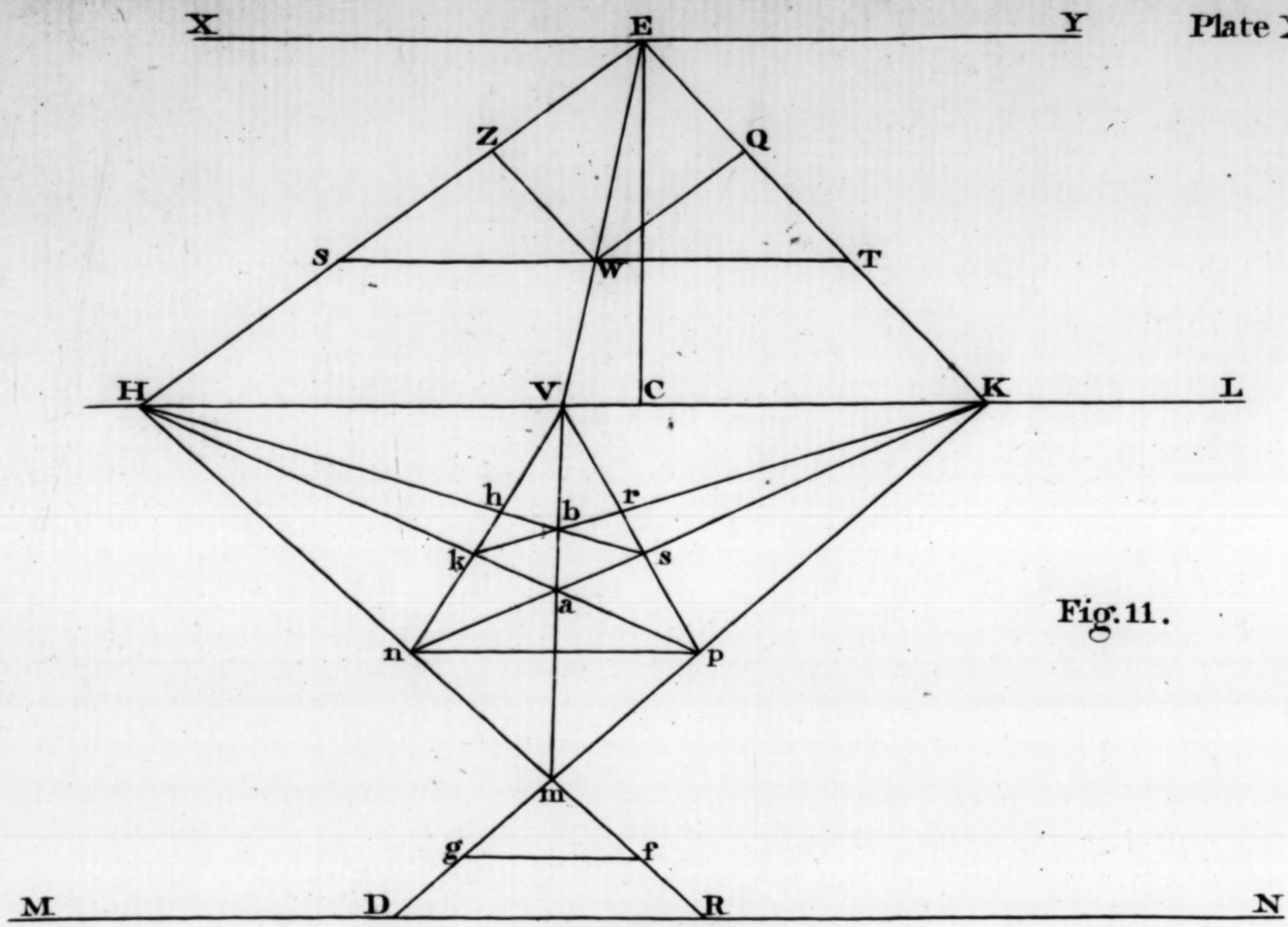

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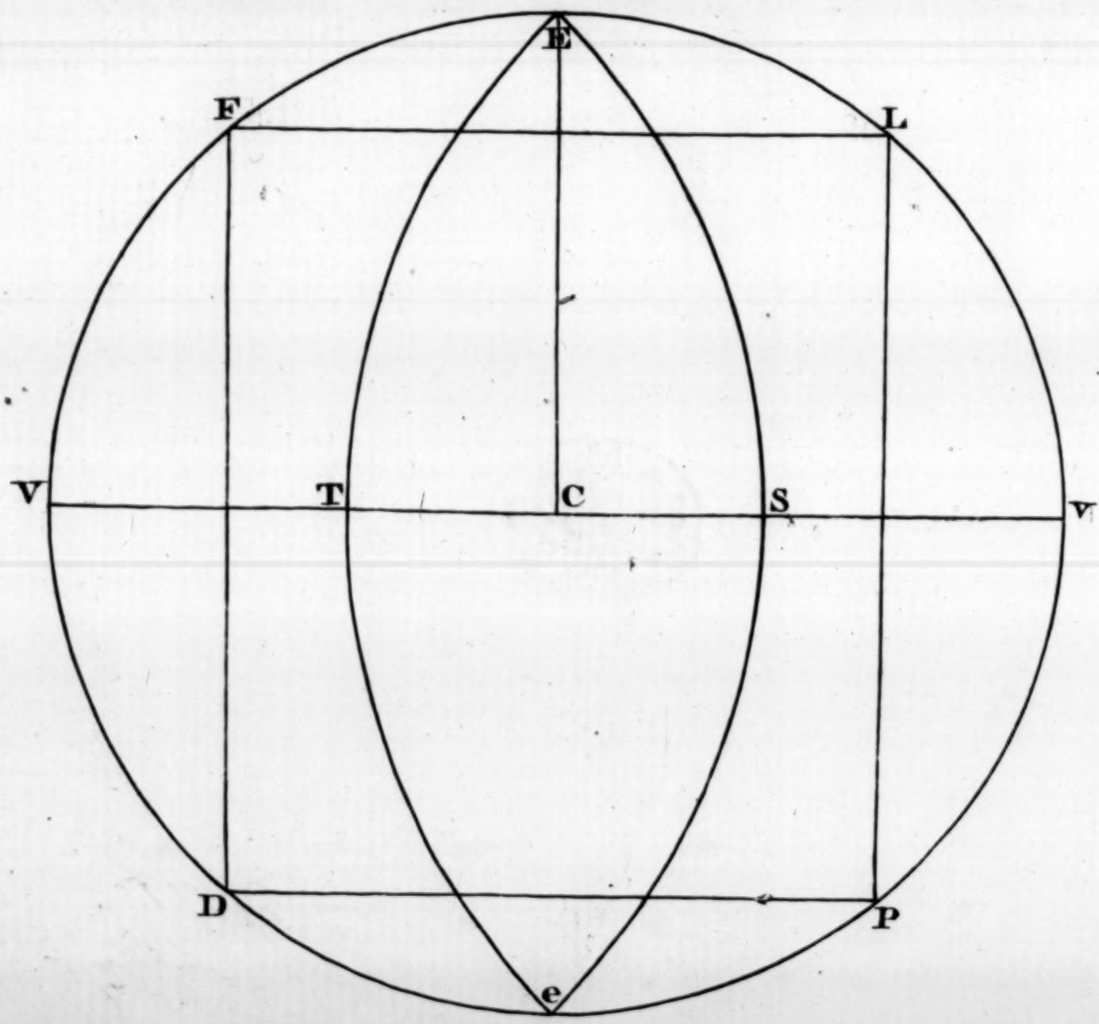


Fig. 12.

Plate XI.

Fig. 13.

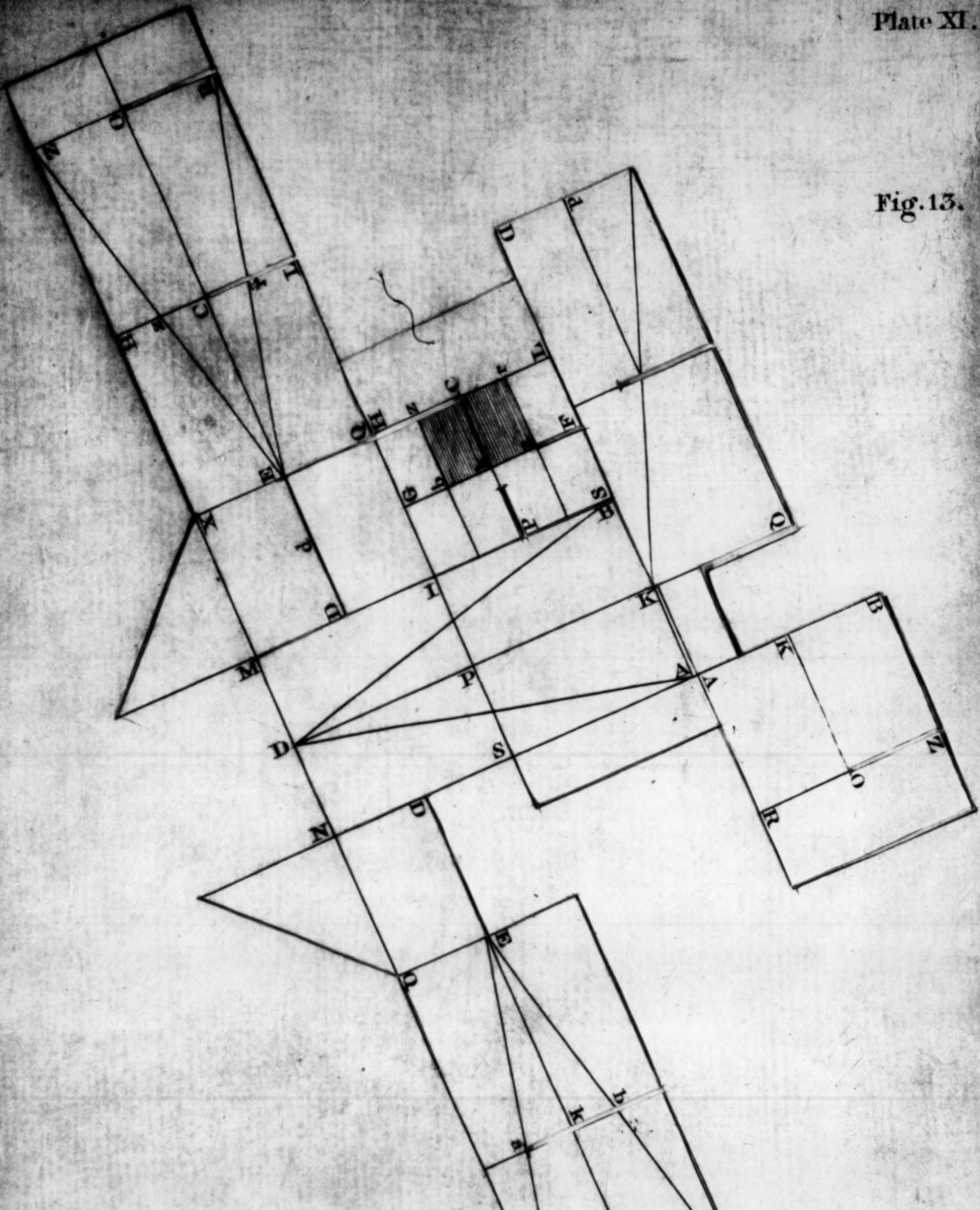
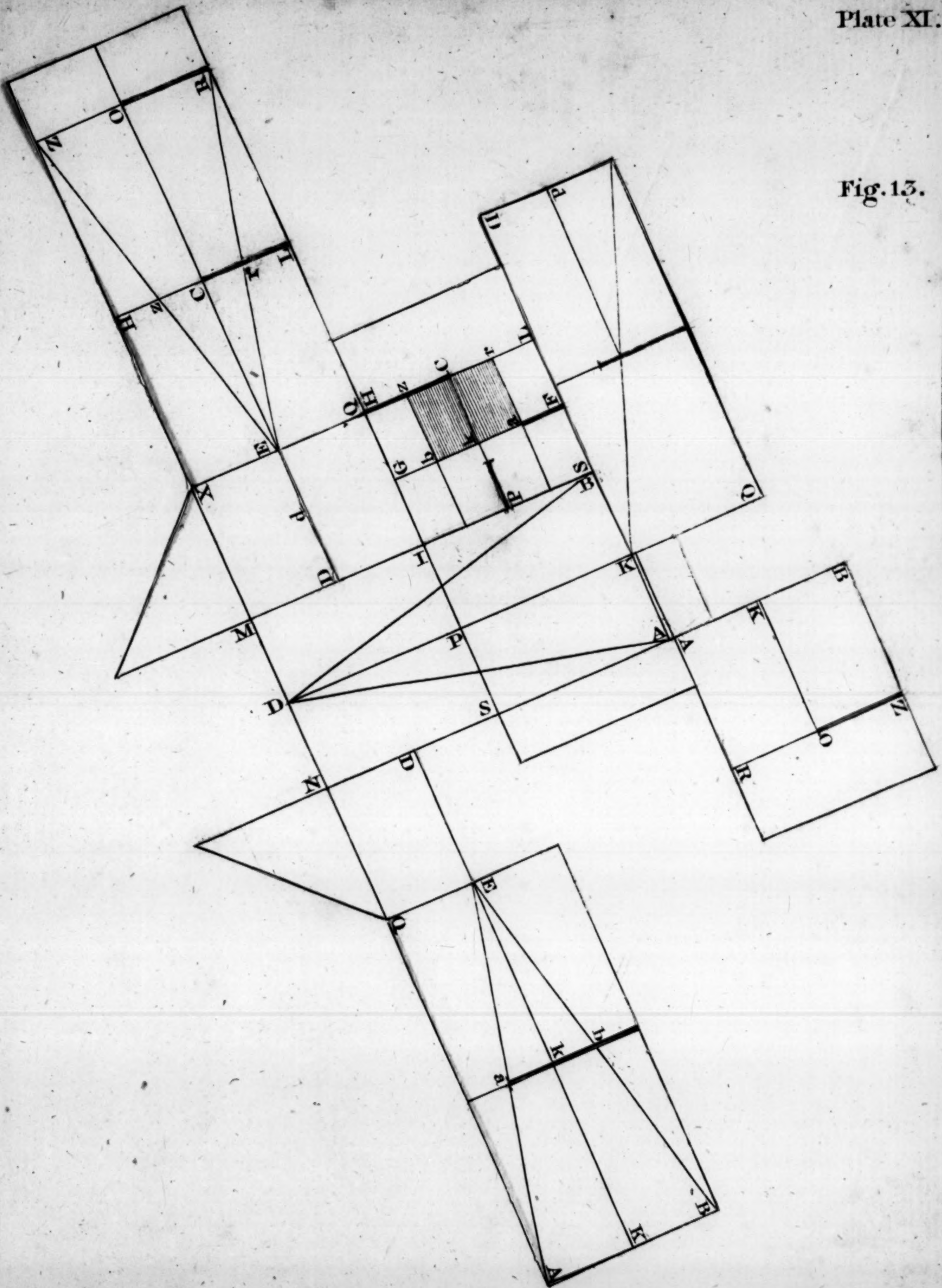
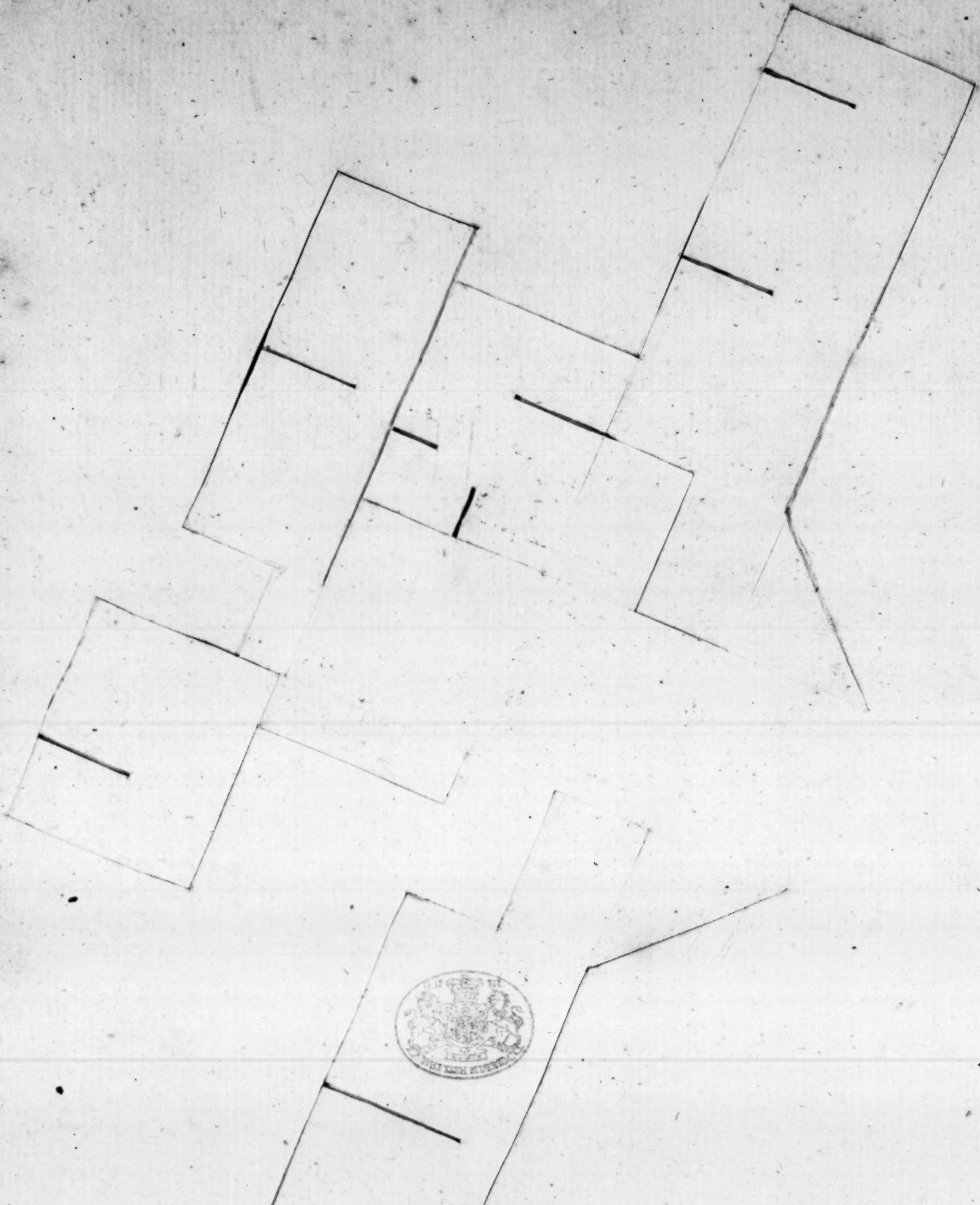


Plate XI.

Fig. 13.





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